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HARVEY'S ESSENTIALS OF ARITHMETIC

SECOND BOOK
PART II

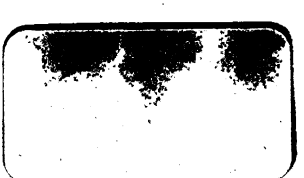


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HARVEY'S ESSENTIALS OF ARITHMETIC

With Everyday Problems Relating to Agriculture, Commerce, and Other Vocations

SECOND BOOK

PART II

BY

L. D. HARVEY, PH.D.

PRESIDENT OF STOUT INSTITUTE, MENOMONIE, WISCONSIN



AMERICAN BOOK COMPANY

NEW YORK CINCINNATI CHICAGO

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HARVEY'S ESSEN. AR. — SECOND BOOK, PART II
E. P. I

PREFACE

THIS book is intended to cover, in particular, the work of the seventh and eighth years. It begins, however, with a brief review of the work through decimals. Great care has been taken to eliminate useless and obsolete subjects and to fill their places with matters relating to the present-day interests of the pupils and to modern business problems.

The main purpose of the book is to secure accuracy and speed in the handling of numbers, and sureness in the interpretation of such problems as come within the range of the pupils' experience. There is thorough drill in abstract number processes. In their concrete application an effort has been made to substitute for unreal or unstimulating conditions many problems of a prevocational character relating directly to the pupils' experience, such as problems in manual training and household arts, commerce, agriculture, etc.

Many problems suggested by recent national legislation have also been included, such as problems relating to Postal Savings Banks, the Federal Income Tax, the Tariff of 1913, and the new Federal Banking law.

The reviews throughout the book are cumulative and thorough. They insure the pupil against forgetting what he has learned, while his energies are concentrated on the mastery of a new process.

Attention is also called to the following features:

1. The work is carefully graded. While the exercises increase gradually in difficulty, and while they are strong enough to test the pupils' power, they are not so hard as to cause discouragement or fatigue.

2. Great emphasis is placed on correct interpretation of problems and on analysis of the reasoning involved. Pupils are, furthermore, encouraged to study problems with the idea of choosing the most economical method of solution, and are advised to make mental estimates, whenever practicable, to be compared with the results of their written work.

3. The chapter on the "Use of Symbols" tests the pupils' power of reasoning and serves as an introduction to the study of algebra.

4. Suggestions to teachers are scattered throughout the book wherever they may serve useful purposes.

5. The importance of self-activity is recognized in asking the pupils to frame problems for themselves and to draw figures that help toward the comprehension of number relations. The "Problems without Numbers" encourage the pupils to construct their own formulas for the solution of various types of problems. They serve as a further aid in the development of the power of reasoning and give the pupils practice in generalization, which will prove invaluable in later mathematical work.

SUGGESTIONS

BEFORE beginning any recitation, the teacher should bear in mind the following four fundamental propositions:

(1) Each lesson should have a definite purpose.

(2) The teacher must clearly realize what must be known and done to accomplish this purpose.

(3) The teacher must consider how much of this material the pupil has already mastered.

(4) The teacher must then determine what the pupil has still to learn and how the known may best be related to the unknown.

The teacher should be sure that the meaning of new and unusual words and expressions is understood by the pupils before they undertake the mastery of the lesson.

A new topic is frequently introduced by what is called the "Study Recitation." The matter under this heading should be mastered by the pupils under the immediate direction and stimulation of the teacher. The teacher should always thoroughly master the treatment of the subject matter under the "Study Recitation" in advance, and should be able to give the illustrations and applications without reference to the book, and to extend or modify them as may be found necessary. These exercises are given where special preparation for further work by the pupils is deemed necessary. The time required for such preparatory work is time saved, even if an entire recitation period is necessary, because the pupils will do the following assigned work more

intelligently, with greater interest, and more rapidly than would be possible without such preparation.

At the close of the most important subjects, a set of review questions is given. As the pupils proceed in their work upon a topic it is an excellent plan for the teacher to assign as a part of the lesson such of these questions as have been covered by the work already done. In this way pupils will be required to prepare upon not more than one or two questions a day, and will need but little time, when the subject is finished, to prepare for a complete summary of it.

Training in the statement of arithmetical facts, conditions, operations, and relations, correct in matter and form, should be carried on throughout the study of the subject, the requirements in this respect being adapted to the pupils' capacity. Definitions are statements of fact, and rules are statements of processes and of operations involving these processes. Unless the pupil can improve upon the form of these statements, he should learn them and make them a part of his arithmetical vocabulary.

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GENERAL REVIEW

Time yourself with these exercises. Then work them again and see whether you can beat your record.

Write from dictation and add. Test by adding in reverse order:

	a.	*b.*	*c.*	*d.*	*e.*
1.	15698	86754	97685	$230.52	$143.28
	54272	70921	74654	$691.04	$876.52
	38314	61832	83732	$570.00	$500.09
	57629	52743	62810	$430.95	$605.98
	14244	43654	51902	$823.74	$721.34

Subtract and test:

2.	43215	29164	48000	40050	60309
	28679	16478	34707	21236	59345
3.	$871.11	$700.00	$500.00	$1596.50	$1428.32
	$654.32	$354.89	$498.95	$ 759.28	$ 649.25

Multiply and test:

4.	5437	8025	6039	7542	8138
	1297	1602	2008	3050	7095
5.	$162.55	$137.89	$200.50	$305.75	$890.60
	1209	1056	3057	2187	6134

Divide and test:

6.	89667 by 123.	124609 ÷ 353.	$1578.28 by 187.
7.	260967 by 301.	163620 ÷ 405.	$7238.40 by 104.
8.	143812 by 229.	642720 ÷ 309.	$1971.84 by 948.
9.	449460 by 198.	104624 ÷ 503.	$8989.20 by 227.
10.	988036 by 994.	152934 ÷ 718.	$3544.92 by 687.

Change to lowest terms:

11. $\frac{18}{27}$ $\frac{30}{45}$ $\frac{25}{100}$ $\frac{48}{90}$ $\frac{18}{48}$ $\frac{45}{72}$ $\frac{27}{36}$ $\frac{16}{24}$

12. $\frac{25}{60}$ $\frac{24}{32}$ $\frac{12}{64}$ $\frac{20}{25}$ $\frac{56}{64}$ $\frac{49}{56}$ $\frac{32}{48}$ $\frac{26}{28}$

Change to improper fractions:

13. $5\frac{3}{4}$ $16\frac{2}{3}$ $37\frac{1}{2}$ $62\frac{1}{2}$ $10\frac{7}{8}$ $15\frac{2}{3}$ $12\frac{3}{8}$ $15\frac{3}{5}$

Change to similar fractions, having the l. c. d. and add or subtract as indicated:

	a.	*b.*	*c.*	*d.*
14.	$\frac{2}{3}+\frac{1}{2}$	$\frac{1}{3}+\frac{3}{4}+\frac{3}{8}$	$\frac{7}{8}-\frac{3}{16}$	$\frac{9}{16}-\frac{3}{8}$
	$\frac{3}{4}+\frac{7}{8}$	$\frac{2}{5}+\frac{3}{4}+\frac{3}{16}$	$\frac{3}{4}-\frac{5}{12}$	$\frac{7}{15}-\frac{2}{5}$
	$\frac{3}{5}+\frac{2}{15}$	$\frac{3}{8}+\frac{3}{8}+\frac{4}{5}$	$\frac{5}{6}-\frac{3}{8}$	$\frac{4}{5}-\frac{3}{4}$

First add; then subtract:

15.	$35\frac{1}{8}$	$74\frac{2}{3}$	$83\frac{1}{4}$	$42\frac{2}{3}$	$79\frac{7}{10}$
	$16\frac{3}{4}$	$35\frac{5}{6}$	$56\frac{5}{8}$	$16\frac{5}{6}$	$35\frac{4}{5}$
16.	$98\frac{5}{6}$	$56\frac{1}{2}$	$72\frac{3}{4}$	$60\frac{1}{2}$	$50\frac{1}{2}$
	$34\frac{2}{3}$	$42\frac{1}{3}$	$67\frac{7}{8}$	$58\frac{4}{5}$	$39\frac{3}{10}$

	Multiply, canceling when possible:		Divide:	
17.	$\frac{3}{4}\times\frac{5}{6}$.	$4\frac{3}{5}\times 1\frac{2}{3}$.	$\frac{7}{8}$ by 5.	$3\frac{7}{8}$ by $2\frac{1}{2}$.
18.	$16\times\frac{5}{8}$.	$6\frac{3}{4}\times 12\frac{3}{8}$.	5 by $\frac{7}{8}$.	$19\frac{3}{4}$ by $3\frac{7}{8}$.
19.	$\frac{2}{3}$ of $\frac{7}{10}$.	$7\frac{1}{2}\times 15\frac{3}{8}$.	$\frac{2}{3}$ by $\frac{4}{5}$.	$25\frac{7}{8}$ by $4\frac{1}{4}$.

Write from dictation and add; then subtract:

20.	356.005	421.	87.057	521.4
	195.3674	16.574	1.3462	413.205
21.	524.0082	209.04	957.	875.05
	315.4367	93.675	642.035	276.945

22. Add the four lines in examples 20 and 21, as though they were one example.

Find products or quotients, as indicated:

	a.	*b.*	*c.*
23.	$750 \times .037$	$.0572 \times 31.257$	$7.182 \div .057$
24.	$.456 \times .789$	1.256×4.009	$10.3707 \div 1.503$
25.	$.0234 \times 1.235$	$7.14 \times .0562$	$38.4804 \div 3.054$
26.	$1.054 \times .0274$	$154 \div .25$	$8113.03 \div 12.109$
27.	25.125×5.08	$26.18 \div 1.54$	$9072.144 \div 3.012$

PROBLEMS

WRITTEN

Farm Problems

1. The annual wheat crop of the United States is about 500,000,000 bushels. If this could be increased .02 by seed selection, how much would the addition be worth at 80¢ per bushel?

2. How many agricultural experiment stations would this increase support at $500,000 a year for each station?

3. If grass seed costs 10¢ a pound and is .20 weed seed, what is the real cost of clean grass seed? How much cheaper per pound would it be to buy a clean seed at 12¢ a pound?

4. If 4 days extra at $2 a day were spent each year in selecting, caring for, and testing seed corn, and the yield of a 40-acre field were thereby increased 10 bushels per acre, how much would be gained, when corn sells at $.71 a bushel.

5. If there are 40 birds per square mile, how many birds are there in the state of Indiana, which has an area of 36,350 square miles?

6. Allowing each bird 50 insects a day for the 120 days of summer, how many insects would be destroyed during the year by 1,454,000 birds?

7. Three golden-winged woodpeckers were found by

King to have in their stomachs, respectively, 255 ants, 220 ants, and 200 ants. Suppose this to represent 1 day's ration, how many ants would these 3 birds eat in 120 days?

8. During an outbreak of Rocky Mountain locusts, a marsh wren was seen to carry 30 locusts to her young in 1 hour. At this rate, how many locusts would she carry in 7 hours?

9. If there are 20 broods of marsh wrens to the square mile, how many locusts would be destroyed in 7 hours in Wisconsin, which has an area of 56,040 square miles, at the rate of 30 locusts per hour for each brood?

10. If each of the 235,368,000 locusts were to consume its own weight of standing crops each day, how much would be saved by the birds, allowing $10 per ton for the crops? Each locust weighs 15 grains, and there are 7000 grains in a pound.

11. If by extra cultivation a man can raise 100 bushels of potatoes more an acre, how many days' work would thus be paid for, allowing $3 per day for man and team, if the potatoes bring $.25 per bushel?

12. The Kansas Experiment Station fed 10 calves on skim milk for 126 days, and they gained an average of 2.02 pounds per day. Another lot of 10 calves, fed for an equal length of time on buttermilk, gained 1.79 pounds per day. What was the difference in total gain between the two lots?

Cooking Problems

Written

13. If the cost of milk is 3.9¢ per pound, and the cost of eggs is 13.3¢ per pound, and the fuel value is 290 and 590 heat units per pound, respectively, what would be the comparative fuel value in 10¢ worth of each?

SOLUTION

10¢ ÷ 3.9 = 2.6, or number of pounds of milk to be obtained for 10¢.
10¢ ÷ 13.3 = .75, or number of pounds of eggs to be obtained for 10¢.
290 × 2.6 = 754, fuel value of 10¢ worth of milk.
590 × .75 = 443, fuel value of 10¢ worth of eggs.
754 ÷ 443 = 1.7 times as much fuel value in 10¢ worth of milk as in 10¢ worth of eggs.

14. Mary wishes to make gingerbread. How much of each ingredient should she use for $\frac{1}{2}$ the following recipe? for $\frac{1}{4}$ of it?

½ cup butter	1 level teaspoonful cinnamon
½ cup sugar	1 level teaspoonful ginger
½ cup New Orleans molasses	1 level teaspoonful cloves
2½ cups flour	1 cup boiling water
2½ level teaspoonfuls soda	2 eggs

½ teaspoonful salt

15. At current prices in your locality, how much would it cost to make the gingerbread from the above recipe?

16. A loaf of bread weighs 1 pound. If the sugar, salt, butter, etc., weigh 4 ounces, and the rest is the weight of the flour, how many cups of flour are used in the loaf?

16 ounces = 1 pound
4 cups of flour = 1 pound

17. If butter is \$.30 per pound; sugar, \$1 per 20 pounds; flour, \$1.50 per 50 pounds; eggs, \$.25 per dozen; 2 teaspoonfuls baking powder, \$.00⅔; and 1 teaspoonful spices, \$.00⅓, determine the cost of the cookies in the following recipe:

1 cup butter	4 eggs
2 cups sugar	2 teaspoonfuls baking powder
3 cups flour	1 teaspoonful spices

2 cups butter = 1 pound
2 cups sugar = 1 pound
4 cups flour = 1 pound

REDUCTION OF DENOMINATE NUMBERS

STUDY RECITATION

Reduction of denominate numbers is the process of changing the denomination without changing the value.

Reduction descending is changing to a number of lower denomination.

1. Reduce 8 gal. 3 qt. 1 pt. to pints.

gal.	qt.	pt.
8	3	1
4		
32		
3		
35 (qt.)		
2		
70		
1		
71 (pt.)		

As 1 gal. = 4 qt., 8 gal. = 8 × 4 qt., or 32 qt. 32 qt. + 3 qt. = 35 qt. As 1 qt. = 2 pt., 35 qt. = 35 × 2 pt., or 70 pt; 70 pt. + 1 pt. = 71 pt.

In practice the work is done as shown at the left, since either factor may be used as multiplier if the numbers are regarded as abstract.

Multiply the given number of units of the highest denomination by the number of units of the next lower denomination it takes to make one of the highest denomination, and to the product add the number of units of the lower denomination, if any. Proceed in like manner with this result and each successive result thus obtained, until the number is reduced to the denomination required.

2. Reduce 11 rd. 4 yd. to yards.

rd.	yd.
11	4
	$5\frac{1}{2}$
	$5\frac{1}{2}$
55	
$60\frac{1}{2}$ (yd.)	
4	
$64\frac{1}{2}$ (yd.)	

If it is required to reduce the $\frac{1}{2}$ yd. to lower denominations, it may be reduced by multiplication as though it were a whole number.

$\frac{1}{2}$
3
$1\frac{1}{2}$ (ft.)

$\frac{1}{2}$ ft. = $\frac{1}{2}$ × 12 in. = 6 in.

The entire result is, 64 yd. 1 ft. 6 in.

Reduce to the next lower unit, or to the lower of the two units mentioned:

	a.	*b.*	*c.*	*d.*	*e.*
3.	5 bu.	6 cwt.	5 score	7 yd.	5 cu. yd.
4.	4 pt.	10 lb.	10 gross	6 ft.	10 cu. ft.
5.	8 qt.	5 yr.	6 doz.	2 A.	5 cd.
6.	6 gal.	10 wk.	6 quires	12 sq. rd.	2 ft. 3 in.
7.	6 T.	10 da.	2 mi.	5 sq. yd.	2 hr. 20 min.
8.	2 L. T.	12 min.	4 rods	10 sq. ft.	12 sq. rd. 6 sq. yd.

9. How many sheets of paper are there in $2\frac{1}{2}$ quires?

10. If from a gross of pencils $7\frac{1}{2}$ doz. are sold, how many pencils are left?

11. A horse is 14 hands in height. What is the difference between his height and yours?

12. A rug is 2 yd. wide and 3 yd. long. How many square yards are there in the surface of the rug? how many square feet?

13. A grocer buys 5 bu. of apples at \$1 per bushel and sells them at 30¢ per peck. How much does he gain?

Written

Change the numbers in column *a* to the denominations in column *b*:

	a. Number	*b.* Denomination		*a.* Number	*b.* Denomination
14.	25 da. 6 hr.	hours	**20.**	12 hr. 35 min. 18 sec.	seconds
15.	2 hr. 30 min.	minutes	**21.**	15 lb. 8 oz.	ounces
16.	15 min. 20 sec.	seconds	**22.**	7 lb. 8 oz.	ounces
17.	52 wk. 1 da.	days	**23.**	8 T. 750 lb.	pounds
18.	24 wk. 6 da.	days	**24.**	11 da. 14 hr.	hours
19.	5 da. 15 hr. 20 min.	minutes	**25.**	4 T. 500 lb.	pounds

	a. Number	*b.* Denomination		*a.* Number	*b.* Denomination
26.	18 ft. 6 in.	inches	**32.**	320 rd. 3 yd.	yards
27.	20 yd. 2 ft.	feet	**33.**	12 rd. 8 ft.	feet
28.	16 rd. 2 yd.	yards	**34.**	24 rd. 12 ft.	feet
29.	28 rd. 5 yd.	yards	**35.**	7 rd. 10 ft.	feet
30.	9 rd. 2 yd.	yards, feet, inches	**36.**	27 rd. 8 ft.	feet
31.	11 rd. 4 yd.	yards, feet, inches	**37.**	9 rd. 7 ft.	feet, inches

Reduction ascending is changing to a number of higher denomination.

STUDY RECITATION

1. Reduce 277 pt. to higher units.

2)277 (pt.)
8)138 (qt.) 1 (pt.)
4) 17 (pk.) 2 (qt.)
4 (bu.) 1 (pk.)

Since 2 pt. = 1 qt., 277 pt. = 138 qt. 1 pt.
Since 8 qt. = 1 pk., 138 qt. = 17 pk. 2 qt.
Since 4 pk. = 1 bu., 17 pk. = 4 bu. 1 pk.
The last quotient and the several remainders give the answer, 4 bu. 1 pk. 2 qt. 1 pt.

Divide the given number by the number of units it takes of its denomination to make one unit of the next higher denomination, and place the remainder, if any, at the right. Divide the quotient thus obtained and each succeeding quotient in the same manner.

The last quotient, with the several remainders annexed in proper order, will be the answer required.

Reduce to the next higher unit:

	a.	*b.*	*c.*	*d.*
2.	8 pt.	32 oz.	21 da.	180 sec.
3.	16 dry quarts	400 lb.	48 hr.	2 score
4.	8 bu.	4000 lb.	24 mo.	24 doz.
5.	16 liquid quarts	4480 lb.	180 min.	60 quires

6. Reduce 28 in. to feet and inches.

7. Reduce 22 ft. to yards and feet.

8. Reduce 36 qt. to pecks and quarts.

9. Reduce 130 min. to hours and minutes.

10. A piece of board 8 ft. long is cut into 6 in. pieces for an exercise in a manual training class. How many such pieces are cut from the board?

11. A box 6 ft. long, 3 ft. wide, and 8 ft. high, inside measure, is filled with sand. How many cubic feet of sand does it hold?

12. A floor is 12 ft. by 15 ft. How many square yards of carpet are required to cover it?

13. A piece of land is 16 rd. wide and 20 rd. long. How many acres are there in the piece?

14. A room is 12 ft. long, 10 ft. wide, and 9 ft. high. How many square yards are there in the four walls?

15. A farmer sells 5000 lb. of hay at $16 per ton. How much does he receive for the hay?

16. What is the cost of 24 qt. of gasoline at 20¢ per gallon?

17. What is the value of 300 lb. of wheat at 90¢ per bushel?

Written

18. In 57 pt., how many gallons and pints are there?

19. In 235 in., how many yards, feet, and inches are there?

20. In 2475 min., how many days, hours, and minutes are there?

21. Reduce 475 cu. ft. to cubic yards and cubic feet.

22. Reduce 240 qt. to bushels and pecks.

23. Reduce 168 ft. to rods and yards.

24. Reduce 20,460 ft. to miles and rods.

25. Reduce 2560 lb. to hundredweight and pounds.

26. Reduce 75 oz. to pounds and ounces.

27. In 778 pt. how many bushels are there?

28. A farmer's load of hay weighs 2430 lb. How much does he receive for it at $16 per ton?

29. A coal merchant delivers a load of coal weighing 4300 lb. How much is the load worth at $8 per ton?

30. A pile of wood 44 ft. long, 6 ft. high, and 4 ft. wide, is sold for $6 a cord. How much is paid for it?

31. A field 80 rd. long and 60 rd. wide yields 15 bu. of wheat per acre. If the wheat is sold at $1 a bushel, what is its value?

32. What is the cost of painting one side of a tight board fence 60 ft. long and 6 ft. high, if two coats of paint are put on, at a cost of 18¢ per square yard?

33. What is the cost of tinting the walls of a room $13\frac{1}{2}$ ft. by 18 ft., if the room is 9 ft. high, at a cost of 7¢ per square yard?

34. What is the cost of tinting the ceiling of the same room, at 7¢ per square yard?

35. The two sides of a roof are to be shingled. Each side is 15 ft. wide and 40 ft. long. If it takes 900 shingles to cover one square (100 sq. ft.), how many shingles are required to cover the roof?

A denominate fraction expressed either as a **common fraction** or as a **decimal** is reduced to lower denominations in the same way as integral denominate numbers are reduced, viz. by multiplication.

Oral and Written

1. What part of a peck is $\frac{5}{12}$ of a bushel? Or, reduce $\frac{5}{12}$ of a bushel to pecks.

Note. Any number of bushels is reduced to pecks by multiplying by 4. Any fraction of a bushel is reduced to pecks in the same way.

Solution. $\frac{5}{12} \times 4 = \frac{5}{3} = 1\frac{2}{3}$. The answer is $1\frac{2}{3}$ pk.

2. Reduce $\frac{3}{8}$ of a peck to quarts. How are pecks reduced to quarts? What is the result?

3. .75 of a foot equals how many inches? How are feet reduced to inches? Apply the process.

4. Reduce $\frac{7}{8}$ of a gallon to lower denominations.

Solution. $\frac{7}{8} \times 4 = 3\frac{1}{2}$, the number of quarts. $\frac{1}{2}$ qt. reduced to pints $= \frac{1}{2} \times 2 = 1$, the number of pints. The answer is 3 qt. 1 pt.

5. Reduce $\frac{4}{5}$ of a yard to lower denominations.

Solution. $\frac{4}{5} \times 3 = 2\frac{2}{5}$, the number of feet. $\frac{2}{5} \times 12 = 4\frac{4}{5}$, the number of inches. *Ans.* 2 ft. $4\frac{4}{5}$ in.

6. Reduce $\frac{7}{8}$ of a day to hours.

7. In $\frac{3}{4}$ of a cord, how many cubic feet are there?

8. Reduce $\frac{1}{8}$ of a barrel to lower denominations.

9. Reduce $\frac{7}{8}$ of a yard to lower denominations.

10. Reduce $\frac{3}{4}$ of a gallon to quarts.

11. Reduce .7 of a pound to ounces.

$$\begin{array}{r} 16 \\ .7 \\ \hline 11.2 \text{ (oz.)} \end{array}$$

In .7 of a pound there are .7 of 16 oz., or 11.2 oz.

12. What decimal of an hour is .07 of a day? .15 of a day?

13. What decimal of a quart is .05 of a peck? .45 of a peck?

14. Reduce $\frac{3}{4}$ of a quart to the fraction of a gallon. How are quarts reduced to gallons?

SOLUTION. $\frac{3}{4} \div 4 = \frac{3}{16}$, the fraction of a gallon.

15. What part of a gallon is $\frac{3}{5}$ of a pint?

SOLUTION. 1 gal. = 8 pt. $\frac{3}{5} \div 8 = \frac{3}{40}$. Hence, $\frac{3}{5}$ pt. = $\frac{3}{40}$ gal.

16. How can you reduce days to weeks? What decimal of a week is .21 of a day?

17. 30.8 rd. is what decimal of a mile?

18. What part of a gallon is 2 qt. 1 pt.?

SOLUTION

2 qt. 1 pt. = 5 pt.
1 gal. = 8 pt.

The question now becomes, 5 pt. is what part of 8 pt.? *Ans.* $\frac{5}{8}$.

19. What part of a day is 12 hr. 30 min.?

SUGGESTION

12 hr. 30 min. = how many minutes?
1 day = how many minutes?
How may the question now be stated?

20. What part of 2 bu. is 2 pk. 2 qt.?

21. What part of a ton of hay is 1800 lb.? what decimal?

Reduce first to a fraction; then to a decimal of the next higher unit:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
22.	7 oz.	240 rd.	1900 lb.	2 ft.	40 min.	5 in.
23.	120 rd.	192 rd.	1600 lb.	8 hr.	40 sec.	750 lb.

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

STUDY RECITATION

In compound numbers, as in simple numbers, the numbers to be added must be of the *same denomination.*

In adding simple numbers, the sum of the numbers of any denomination is divided by ten, the quotient is added to the next higher denomination, and the remainder is written under the numbers added. In adding compound numbers, the sum of the numbers of any denomination is divided by the number of units of that denomination it takes to make one of the next higher denomination; the remainder, if any, is written under the numbers added, as in simple numbers, and the quotient is added to the number of the next higher denomination, as in simple numbers.

Add:

1.

wk.	da.	hr.
7	6	12
5	4	13
3	5	10
17	2	11

Adding the hours gives the sum 35 hr. Dividing 35, the number of hours, by 24, to reduce to days, gives 1 da. and the remainder 11 hr. Write the remainder, 11, under the hours added. Add the 1 da. to the numbers representing days; the sum is 16 da. Dividing 16, the number of days, by 7, to reduce to weeks, gives 2 wk., with 2 da. remainder. Write the 2 under the days added. Add the 2 wk. to the numbers representing weeks; the sum is 17 wk. The entire sum is 17 wk. 2 da. 11 hr.

2.

rd.	yd.	ft.	in.
8	5		
3	2		
4	3		
16	4	1	6

Adding the yards, gives the sum, 10 yd. Dividing 10, the number of yards, by $5\frac{1}{2}$, to reduce to rods, gives 1 rd., with $4\frac{1}{2}$ yd. remainder. Write the $4\frac{1}{2}$ yd. under the numbers added. Add the 1 rd. to the rods; the sum is 16 rd. The entire sum is 16 rd. $4\frac{1}{2}$ yd.

The $\frac{1}{2}$ yd. may be reduced to lower denominations by multiplication. $\frac{1}{2}$ yd. $= 1\frac{1}{2}$ ft. $\frac{1}{2}$ ft. $= 6$ in. Instead of writing $4\frac{1}{2}$ yd., the sum may be written 4 yd. 1 ft. 6 in., the entire sum being 16 rd. 4 yd. 1 ft. 6 in.

Add:

3.

rd.	ft.
5	14
2	13
8	10
17	4

Adding the feet gives the sum, 37 ft. Divide 37, the number of feet, by $16\frac{1}{2}$, the number of feet in a rod, by reducing $16\frac{1}{2}$ to $\frac{33}{2}$ and 37 to $\frac{74}{2}$. Dividing $\frac{74}{2}$ the number of feet, by $\frac{33}{2}$ gives 2, the number of rods, and $\frac{8}{2}$ ft., or 4 ft., remaining. Write the 4 ft. under the number of feet added, and add the 2 rd. to the rods. The entire sum is 17 rd. 4 ft.

4.

	bu.	pk.	qt.
From	8	2	5
take	5	3	7
	2	2	6

7 qt. cannot be taken from 5 qt. Take 1 pk. from the 2 pk. and reduce it to 8 quarts. 8 qt. + 5 qt. = 13 qt. 7 qt. from 13 qt. = 6 qt. 3 pk. cannot be taken from the 1 pk. remaining. Take 1 bu. from 8 bu. and reduce it to 4 pk. 4 pk. + 1 pk. = 5 pk. 5 pk. − 3 pk. = 2 pk. 5 bu. from the 7 bu. remaining equals 2 bu. The entire answer is 2 bu. 2 pk. 6 qt.

WRITTEN

Add:

5.

mi.	rd.	yd.
2	120	5
2	250	3
1	17	1

6.

mi.	rd.	ft.
1	180	15
2	120	13
3	240	9

7.

bu.	pk.	qt.
8	3	5
25	2	7
16	3	6

8.

T.	lb.
8	420
3	965
4	868

9.

hr.	min.
16	45
12	30
20	12

10.

sq. yd.	sq. ft.
3	7
4	5
7	8

11.

cu. ft.	cu. in.
8	1200
15	1440
19	758

Prove the correctness of answer in each case:

12.

	hr.	min.
From	13	15
take	8	50

13.

	mi.	rd.
From	1	180
take		260

14.

	doz.	units
From	$8\frac{1}{2}$	
take	4	3

15.				16.				17.			
	bu.	pk.	qt.		gal.	qt.			yd.	ft.	in.
From	9	3	2	From	8	2		From	9	2	7
take	4	2	6	take	2	3		take	2	2	9

Finding the difference between dates.

In finding the difference between two dates, treat the number of the year as so many years, the number of the month as so many months, and the day of the month as so many days. In subtracting, count 30 days to the month and 12 months to the year.

1. Find the difference in time between Sept. 8, 1914, and Nov. 21, 1909.

yr.	mo.	da.
1914	9	8
1909	11	21
4	9	17

The later date is written as 1914 yr. 9 mo. 8 da. The earlier date is written as 1909 yr. 11 mo. 21 da. The subtraction is performed in the same way as in other cases of subtraction in compound numbers.

2. A man who was born Nov. 3, 1844, died Jan. 18, 1914. At what age did he die?

3. If you have a brother who is 4 yr. 6 mo. 10 da. older than you are, what was the date of his birth?

4. A man gave a note Aug. 10, 1904, and paid it June 7, 1906. How long did the note run?

Subtract each date from the date above it:

5. 1914, May 12
 1913, Sept. 16

6. 1912, Nov. 4
 1911, Dec. 5

7. 1911, Aug. 6
 1909, Dec. 9

8. 1914, Jan. 4
 1908, April 6

9. 1914, Jan. 1
 1906, May 8

10. 1913, Feb. 2
 1905, Oct. 5

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS

Study Recitation

1. Multiply 3 ft. 8 in. by 4.

ft.	in.
3	8
	4
14	8

4×8 in. $= 32$ in. $= 2$ ft. 8 in. Write the 8 in. under the denomination inch, and add the 2 ft. to the product of 4×3 ft. Write the sum, 14 ft., under the denomination feet. The product is 14 ft. 8 in.

2. Multiply 2 rd. 12 ft. 4 in. by 4.

rd.	ft.	in.
2	12	4
		4
10	16	4

4×4 in. $= 16$ in. $= 1$ ft. 4 in. Write the 4 in. under inches, and add the 1 ft. to the product of 4×12 ft. The sum is 49 ft. Divide 49 ft. by $16\frac{1}{2}$, the number of feet in a rod, by reducing both to half feet.

$16\frac{1}{2}$ ft. $= \frac{33}{2}$ ft. 49 ft. $= \frac{98}{2}$ ft.

$\frac{33}{2}$ ft. is contained in $\frac{98}{2}$ ft. 2 times and $\frac{32}{2}$ ft. remaining. $\frac{32}{2}$ ft. $= 16$ ft. Write the remainder, 16 ft., under the denomination feet, and add 2 rd. to the product of 4×2 rd. The sum is 10 rd. The entire product is 10 rd. 16 ft. 4 in.

3. Multiply 7 rd. 11 ft. 6 in. by 15.

4. What is the product of 25 bu. 3 pk. by 14?

5. Divide 15 bu. 3 pk. by 4.

bu.	pk.	qt.
4)15	3	
3	3	6

$\frac{1}{4}$ of 15 bu. $= 3$ bu., with 3 bu. remaining undivided. 3 bu. $= 12$ pk. 12 pk. $+ 3$ pk. $= 15$ pk. $\frac{1}{4}$ of 15 pk. $= 3$ pk., with 3 pk. remaining undivided. 3 pk. $= 24$ qt. $\frac{1}{4}$ of 24 qt. $= 6$ qt. The quotient is 3 bu. 3 pk. 6 qt.

Written

6. Divide 115 rd. 2 yd. 1 ft. 6 in. by 15.

7. A man has a farm of 160 A. 100 sq. rd., which he wishes to divide into 6 equal lots. What is the area of each lot?

8. A man walks 3 mi. 70 rd. the first hour, 4 mi. 10 rd. the second hour, and 3 mi. 50 rd. the third hour. What is the average distance that he walks each hour?

9. How many pieces of board 1 ft. 3 in. long can be sawed from a board 16 ft. 6 in. long, if $1\frac{1}{2}$ in. are wasted in sawing? How much of the board will be left? Reduce both dimensions to inches and divide.

10. If $12\frac{1}{2}$ lb. of spices are put into 8 packages of equal size, what is the weight of each package?

11. Make a rule for multiplication of compound denominate numbers; for division of compound denominate numbers.

To the Teacher. These exercises need not be given unless further practice is deemed necessary.

Multiply or divide as indicated:

12. 6×1 pk. 3 qt.
13. 5×2 bu. 3 pk.
14. 12×2 lb. 5 oz.
15. 10×1 T. 250 lb.
16. 24×5 yd. 2 ft.
17. 3×5 yd. 1 ft. 6 in.
18. 4×16 ft. 6 in.
19. 12×4 rd. 4 ft.
20. 15×3 qt. 1 pt.
21. 6×4 gal. 3 qt.
22. 8×2 mi. 180 rd.
23. 4×3 sq. yd. 7 sq. ft.
24. 8×1 cu. yd. 5 cu. ft.
25. 5×4 hr. 40 min.
26. 54 mi. 222 rd. 3 yd. $\div$ 9.
27. 93 gal. $\div$ 8.
28. 349 bu. 4 qt. $\div$ 14.
29. 1 mi. 122 rd. 1 yd. $\div$ 12.
30. 3 gal. 1 pt. $\div$ 5.
31. 12 hr. 30 min. $\div$ 10.
32. 5 mi. 240 rd. $\div$ 8.
33. 14 ft. 8 in. $\div$ 11.
34. 10 yd. 2 ft. $\div$ 8.
35. 2 T. 1600 lb. $\div$ 4.
36. 3 cu. yd. 3 cu. ft. $\div$ 6.
37. 6 rd. 9 ft. $\div$ 12.
38. 8 ft. 4 in. $\div$ 2 ft. 1 in.
39. 1 bbl. 3 qt. $\div$ 10 gal. 3 qt.

In examples 38, 39, reduce the dividend and divisor to the same denomination before dividing.

PROBLEMS

WRITTEN

1. If a locomotive consumes an average of 93 lb. 6 oz. of coal per mile, how many tons will it consume on a run of 168 mi.?

2. If each of a span of horses eats 1 pk. 6 qt. of oats a day, and oats are worth 48¢ per bushel, what is the cost of oats fed to a span of horses during the month of December?

3. From an alfalfa field of 15 acres, 3 crops were harvested in one season. The average yield per acre for each crop was 2850 lb. How much was the season's yield worth at $14.50 per ton?

4. A side of bacon weighs 8 lb. 12 oz. A ham weighs 13 lb. 4 oz. What is the cost of both, if bacon is worth 24¢ per pound, and ham, 20¢ per pound?

5. If 1000 shingles are required to cover 100 sq. ft. of roof, how many shingles are required to shingle the two sides of a roof if each side is 40′ by 18′?

6. If shingles are bought in bunches, each bunch containing 250 shingles, how many bunches must be bought for the roof in problem 5? How much are they worth at $3.50 per M?

7. A man owned 32 acres of land. He bought another tract 80 rd. long by 60 rd. wide. He then sold 15 A. 60 sq. rd. How much land had he left?

8. If a ton of timothy hay occupies 500 cu. ft. in a mow, how many tons will a mow hold whose dimensions are 40′ × 18′ × 15′?

9. A dealer buys ½ T. of cheese at $250 per ton. If he sells it at an average price of 16½¢ per pound, what is his gain?

10. A housekeeper can buy ice at 50¢ per C, paying at the end of each month. Paying in advance, she can buy a book of tickets entitling her to 1000 lb. for $4. If she buys two books at the same time, they cost her $7. If she uses $2\frac{1}{2}$ T. of ice during the year, what is the cost by each plan of payment?

11. A farmer sold a load of corn weighing 2748 lb., and a load of wheat weighing 2560 lb. For the wheat he received $.76 a bushel, and for the corn $.55 a bushel. How much did he receive for both?

12. An ocean steamer during a voyage of 3025 mi. burns coal at the rate of 1 T. 95 lb. for every 25 mi. How many tons of coal are used up in the voyage?

13. What is the cost of the following coal bill, at $7.50 per ton: 3650 lb., 2720 lb., 5000 lb., 1960 lb., 2750 lb.?

14. A machinist ships twelve machines weighing 3 T. 750 lb. each. How much does he pay for freight at 22¢ per hundredweight?

15. A milkman sold 25 gal. 3 qt. of cream on Monday, 20 gal. 2 qt. on Tuesday, and 21 gal. 1 qt. on Wednesday, at $1.20 per gallon. How much did he receive for the cream?

16. For each of the remaining 4 days of the week, his sales averaged the same as for each of the first 3 days. How many gallons did he sell during the 4 days?

17. A milkman owns a cow that gives an average of 4 gal. 3 qt. of milk daily during the month of June. How much does he receive for the milk if it is sold for $.07 per quart?

18. A mail carrier travels 92 mi. 80 rd. in delivering mail for 1 week of 6 days. If he makes 2 deliveries daily, how far does he travel in making each delivery?

19. A bicycle wheel 7 ft. 4 in. in circumference makes 720 revolutions in 2 minutes. At what rate per hour does the wheel revolve?

NOTE. The *circumference* is the distance around the wheel.

20. A dealer sold 420 T. 1250 lb. of coal at $5.50 per ton, and a number of tons of other kinds at $7.50 per ton. In all, he received $4780. How many tons of the second kind did he sell?

21. How many steps of 33 inches each must be taken in walking 4 rods?

22. How many steps of 30 in. each must be taken in walking 2 mi. 300 rd.?

23. How many times will a bicycle wheel 88 in. in circumference turn in going one mile?

24. How many times will a bicycle wheel 88 in. in circumference turn in going $12\frac{3}{8}$ miles?

25. How many strips of carpet 30 in. wide are required to cover a floor 20 ft. $\times$ 25 ft., if the strips run lengthwise? crosswise?

26. How many strips of carpet 30 in. wide are required to cover a floor 18 ft. $\times$ 22 ft., if the strips run lengthwise? crosswise?

27. In carpeting the floor in problem 26, if the strips are laid lengthwise, when seven strips have been laid, what is the width of the portion of the floor not carpeted?

In buying carpet what must be bought to cover the remaining portion of the floor? Draw a figure on the board representing the floor on the scale of 1 inch to 1 foot. Draw lines showing the lines of joining of the strips.

28. How many square rods are there in a piece of land 24 rd. long and 12 rd. wide?

29. In 288 sq. rd., how many acres are there?

30. What is the cost of making a macadam road 1 mi. long and 12 ft. wide, at 23¢ per square yard?

31. How much would it cost to dig a cellar 18 ft. long, 15 ft. wide, and 6 ft. deep, at 50¢ per cubic yard?

32. One quart, dry measure, contains $67\frac{1}{5}$ cu. in. One quart, liquid measure, contains $57\frac{3}{4}$ cu. in. One bushel equals 32 qt., dry measure. How many more cubic inches are there in 1 bu. than in 8 gal.?

33. How much can a man earn in 5 days digging a ditch, at the rate of $12\frac{1}{2}$¢ per running foot, if he digs 4 rd. per day?

34. A man wishes to pack 42 bu. of apples in barrels. If each barrel contains 3 bu. 2 pk., how many barrels will it take?

35. If you study 45 min. at home each day your school is in session, how many hours of home study will you do during the school year?

36. If you devote 15 min. a day, each day in the year, to reading good literature, how many hours' reading will you do in 1 year? Omit fractions. See how many pages of a book worth reading you can read in 1 hr., and find the number of pages you can read in 1 yr., reading 15 min. a day.

37. How many feet of wire are required for a 4-wire fence around a field 30 rd. 12 ft. long by 20 rd. 8 ft. in width?

38. Sound travels at the rate of 1120 ft. per second. If the time between a flash of lightning and the sound of thunder is 12 sec., how far distant is the storm? There are 5280 ft. in a mile. Give answer in miles and feet.

39. A coal dealer delivers 4 loads of coal to a customer. The loads weigh, respectively, 2 T. 315 lb., 2 T. 340 lb., 1 T. 1950 lb., 2 T. 325 lb. What is the value of the coal at $7 per ton?

40. How many acres are there in a farm 134 rd. long and 52.5 rd. wide?

41. From a lot containing 15 acres of land, a man sold 1400 sq. rd. How many acres did he have left?

42. Draw a rectangle 6 in. by 8 in. to represent a flower bed 12 ft. by 16 ft. Draw lines to represent a walk 2 ft. wide around the bed. What is the length of the walk on two sides of the bed? What is the length of the remaining parts of the walk on the two ends?

43. How many square feet are there in the flower bed mentioned in problem 42? in the walk?

44. A lot 120 ft. long and 80 ft. wide is surrounded by a sidewalk 8 ft. wide. What will be the cost of the sidewalk at $1.50 per square yard?

45. Draw a rectangle on a scale of 1 in. to 8 ft. to represent the lot. Add lines to show the outer boundary of the sidewalk. Add four lines dividing the sidewalk into four rectangles.

46. A door 7 ft. high and 3 ft. wide has a 6 in. casing around it. How many square inches are there in the surface of the casing?

47. Make a drawing on a scale of 1 in. to 1 ft. to represent the door and casing whose dimensions are given in problem 46. If the stock for the casing is cut from two boards 8″ wide, one 10′ long, and the other 12′ long, how many board feet of 2″ lumber are required? How many board feet are wasted?

48. If a rug 12 ft. long and 10 ft. wide is laid on a floor so as to leave a margin of 2 ft. in width all around the rug, what is the area of the floor? What is the area of the part not covered by the rug? Draw a plan of the floor and the rug on a scale of 1 in. to 1 ft.

49. From a lot of land 80 rd. square, 80 sq. rd. were sold. What was the value of the remainder at $50 per acre?

50. A pasteboard box is 8 in. long, 6 in. wide, and 4 in. high. How many square inches of pasteboard will it take to make the sides of the box? to make the ends? to make the top and bottom? How much more than a square foot of pasteboard will be required?

51. A box is 8 in. long, 6 in. wide, and 4 in. high. What is the length and the width of one piece of paper that will just cover the sides and the ends of the box? Draw a rectangle on a scale of 1 in. to 8 in., to represent the piece of paper. Divide the figure by three lines drawn across it, making four rectangles to represent the sides and ends of the box.

52. Draw a rectangle on the same scale to represent the top and the bottom of the box. Write the correct dimensions on each rectangle drawn. Find the area of the paper necessary to cover the box.

53. A room is 18 ft. long, 15 ft. wide, and 9 ft. high. How many square yards are there in the walls, floor, and ceiling of the room?

54. Draw a rectangle on a scale of 1 in. to 6 ft. to represent the four walls of the room described in problem 53, if placed end to end. Draw another rectangle to represent the floor and the ceiling.

55. A city lot is 30 ft. front by 60 ft. deep. If it sells at $5 per square foot, how much is the lot worth? If it sells at $75 per front foot, how much is it worth?

56. A garden 40 ft. by 60 ft. has a gravel walk 4 ft. wide around it. How many square feet are there in the walk? Illustrate by a diagram.

57. A room is 15 ft. by 18 ft. How many square yards of plastering are there in the ceiling? How many yards of carpeting $\frac{3}{4}$ of a yard wide will it take to cover the floor? Which way should the carpet be laid?

58. A wall is 20 ft. long and 10 ft. 6 in. high, with an 18-in. baseboard. How much will it cost to plaster the wall at 12¢ a square yard? If this wall is one side of a room 16 ft. wide, how much will it cost to plaster the four walls and the ceiling at the same rate, making no allowance for doors or windows?

59. How many tiles 6 in. by 4 in. will it take to cover the floor of a hall 8 ft. by 6 ft.?

60. How much glass is there in 12 windows of 8 panes each, if each pane contains 1 sq. ft. 48 sq. in.?

61. How many feet of lumber are there in ten boards 14′ long, 6″ wide, and 1″ thick?

62. How many feet of lumber are there in nine joists 18′ long, 8″ wide, and 2″ thick?

63. Find the cost at $24 per M of 17 boards 16′ long, 10″ wide, and 1″ thick; of 8 scantlings 14′ long, 4″ wide, and 2″ thick; and of 18 joists 12′ long, 6″ wide, and $2\frac{1}{2}$″ thick. Put this in the form of a receipted bill.

64. How much will it cost for the flooring for a floor 21 ft. by 16 ft. at $25 per M, allowing nothing for waste?

The flooring is supposed to be 6 in. in width, and the strips are tongued and grooved.

If the strips are tongued and grooved, the number of board feet of lumber required is $1\frac{1}{12}$ times the number of square feet in the surface of the floor.

65. If a thousand shingles are needed to cover 100 sq. ft. of roof, how many thousand shingles are required to cover a gable roof 50 ft. long, each of the two sides being $22\frac{1}{2}$ ft. wide? What will be the cost of the shingles at \$4 per M?

66. Measure a door in the schoolroom and the width of its casing, and find the number of square feet in the surface of the door and in the surface of the casing.

67. What will be the cost of cork matting to cover the floor of your schoolroom at 90¢ per square yard?

68. A room 20 ft. by 18 ft. is to be covered with carpet 27 in. wide, worth \$1.50 per yard. After the first strip, it is necessary to cut off 9 in. from each of the remaining strips in order to match the figures. How much will it cost to carpet the room?

69. How many square feet are there in a floor 14 ft. 7 in. wide and 19 ft. 4 in. long?

70. At $12\frac{1}{2}$¢ a square yard, how much will it cost to sod a lawn 40 ft. long by 36 ft. wide?

71. A string 40 ft. long was used to outline a square flower bed, the string reaching entirely around it. What were the dimensions of the bed? What was its area? If the bed had the same area and was 20 ft. long, what length of string would be required to go around it? Draw figures on a scale of 1 in. to 10 ft. for both beds.

72. The distance between two cities is 55 miles. How many times will an engine wheel 18 ft. 6 in. in circumference turn in making the trip?

73. From a cask of oil containing 33 gal., 18 gal. 3 qt. were drawn. How much remained?

74. How many baskets, each holding 3 pk., can be filled with 12 bu. of apples?

75. How many ties 8 in. wide will be laid on $20\frac{2}{3}$ ft. of a railroad, if the ties are laid 2 ft. from center to center? Draw a figure on a scale of 1 in. to 2 ft. showing that your answer is correct. How many ties will there be on $24\frac{2}{3}$ ft. of the road?

76. At \$.40 per tie, how much will it cost for 2640 railroad ties for one mile of a road?

77. How many barrels, each holding 2 bu. 3 pk., will it take to hold 100 bu. of apples? How many pecks will there be in the partly filled barrel?

78. If it requires 12 qt. of oats per day to feed a horse, how many days will 6 bu. 3 pk. last a span of horses?

79. If a train travels 85 miles in 2 hr. 30 min., what is its rate per hour?

80. At \$7 a cord, what is the cost of a load of wood 12 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. high?

81. A farmer sells 3 loads of hay, each containing 1785 lb. If he sells it at \$7 per ton, how much does he receive for it?

82. If 12 equal bins hold 430 bu. 2 pk. of wheat, how much wheat is there in each bin?

83. How many bushels of wheat are there in 12 bins, each containing 124 bu. 3 pk. 5 qt.?

84. Reduce 2 pk. 4 qt. to the decimal of a bushel.

85. If 9 bu. 2 pk. of potatoes cost \$3.80, what is the average cost per bushel? per peck?

86. If a school uses 24 crayons a week, it will use how many gross of crayons in 36 weeks?

87. If a dealer buys pencils at \$3.50 per gross and sells them at 5¢ apiece, how much does he gain on each gross? on 30 dozen?

88. A tank 2 ft. wide, 4 ft. high, and 6 ft. long will contain how many gallons? how many bushels?

89. A cube is 4 in. square. If all the sides are covered with paper, how many square inches of paper are required?

90. How many cubic yards of earth will be taken out in digging a cellar 20 ft. long, 12 ft. wide, and 5 ft. 4 in. deep?

91. How many bushels of oats does a bin 8 ft. by 6 ft. by 5 ft. contain? How much do they weigh?

92. During a heavy rain, 2 in. of water fell. How many gallons of water fell on a garden 60 ft. by 80 ft.?

93. If a schoolroom is 24 ft. wide, 36 ft. long, and 10 ft. high, how many cubic feet of space will there be to each of 40 pupils?

94. How many feet of boards are required for a coal bin in the corner of a cellar, so constructed that the walls of the cellar form two sides of the bin, and the floor of the cellar the bottom of the bin, the bin being 12 ft. long, 6 ft. wide, and 8 ft. high? (Boards 1 in. thick.)

95. A cask holds 52 gallons of water. The contents will fill how many bottles, if it requires 5 bottles to hold one gallon of water?

96. Sound travels 1120 ft. per second. A hunter sees the flash of a gun at a distance, and 4 seconds later hears the report. What fraction of a mile distant is he from the gun?

97. When shingles are laid 4 in. to the weather, how many shingles will cover the two sides of a roof, each side being 40 ft. by 20 ft.?

NOTE. Shingles are 16 in. long and of different widths, but in reckoning, each width of 4 in. is considered one shingle, and a bundle of shingles is considered as made up of 250 shingles 16 in. long and 4 in. wide. If a shingle is laid 4 in. to the weather, then each shingle will cover 16 sq. in. of space.

98. If a shingle is laid $\frac{1}{4}$ to the weather, how many shingles does it take to cover a square foot? When the length and width of a roof are given, and shingles are laid $\frac{1}{4}$ to the weather, what is the process of finding the number of shingles required to cover the roof? How can you find the number of bundles of shingles required to cover the roof?

NOTE. To the exact number of shingles required to cover 1 square, or 100 sq. ft., about 100 are added to allow for waste in laying.

REVIEW

1. State the difference between a simple denominate number and a compound denominate number. How may the latter be changed to the former?

2. Define quantity; measure. What is meant by measuring a quantity?

3. Define surface; angle; right angle. Give an example of each.

4. What is the difference between a square and a rectangle?

5. How many dimensions has a line? a square? a rectangular solid?

6. What is a cube? a rectangular solid? Give an example of a rectangular solid that is not a cube.

7. 2 in. is what fraction of 1 ft.? what decimal of 1 ft.?

8. 3 qt. is what fraction of 1 gal.? what decimal of 1 gal.?

9. 25 min. is what fraction of 1 hr.? what decimal of 1 hr.?

10. Give each of the tables of denominate numbers, its use, and the abbreviation used for each denominate unit.

Problems without Numbers

11. How can you find the area of a rectangle when its length and breadth are known? Illustrate by an example.

12. When one dimension and the area of a rectangle are given, how can you find the other dimension? Illustrate by an example.

13. How can you find the volume of any rectangular solid? Illustrate by an example.

14. Having two dimensions and the volume of a rectangular solid, how can you find the other dimension? Illustrate by an example.

15. The dimensions of a square, A, are one half the dimensions of a square, B; the surface of A is what part of the surface of B? Illustrate by a diagram.

16. If the dimensions of a square, A, are one third the dimensions of a square, B, the area of B is how many times the area of A? Illustrate by a diagram.

17. Having given the dimensions of a rectangular room in feet, how can you find the area of the four walls, making but one multiplication? making two multiplications? making four multiplications? What other operation is necessary in each case? Illustrate by an example.

18. Having given any number of units of one denomination, how can you find what fraction they are of one unit of the next higher denomination? Illustrate by original examples.

19. Having given any number of units of one denomination, how can you find what decimal they are of one unit of the next higher denomination?

20. Give original examples under each table, to illustrate reduction ascending, reduction descending.

RATIO AND PROPORTION

RATIO

Study Recitation

The **ratio** of one number to another of the same kind is the quotient arising from the division of the first number by the second.

The ratio of 4 to 2 is $4 \div 2$, or 2; the ratio of 10 to 2 is $10 \div 2$, or 5; the ratio of 5 to 3 is $5 \div 3$, or $\frac{5}{3}$; the ratio of 7 to 9 is $7 \div 9$, or $\frac{7}{9}$; the ratio of $\frac{5}{6}$ to $\frac{2}{3}$ is $\frac{5}{6} \div \frac{2}{3}$, or $\frac{5}{4}$.

The ratio of two numbers may be expressed by writing a colon between them; as,

$6 : 9$, read, the ratio of 6 to 9.
$4 : 2$, read, the ratio of 4 to 2.
$\frac{1}{3} : \frac{3}{4}$, read, the ratio of $\frac{1}{3}$ to $\frac{3}{4}$.

The **antecedent** is the first term of a ratio.

The **consequent** is the second term of a ratio.

A ratio may always be expressed as a fraction, the numerator being the antecedent, and the denominator, the consequent.

The ratio of $6 : 7 = \frac{6}{7}$ $\begin{matrix}\text{Antecedent.}\\ \text{Consequent.}\end{matrix}$

A fraction is the ratio of the numerator to the denominator. Since every ratio may be regarded as a fraction, the laws governing the changes of one or both terms of a fraction by multiplication or division apply to changes in the terms of a ratio.

Multiplying the (numerator) of a (fraction) multiplies the (fraction). Dividing the (numerator) of a (fraction) divides the (fraction). Multiplying the (denominator) of a (fraction) divides the (fraction). Dividing the (denominator) of a (fraction) multiplies the (fraction).

Multiplying or dividing both (numerator) and (denominator) of a (fraction) by the same number does not change the value of the (fraction).

In the foregoing statements, substitute the word "antecedent" for the word "numerator," the word "consequent" for the word "denominator," and the word "ratio" for the word "fraction," and give each statement as thus changed.

Multiplying or dividing both terms of a ratio by the same number does not change its value.

EXAMPLES AND PROBLEMS

ORAL

Find the ratio of:

	a.	*b.*	*c.*	*d.*
1.	16 to 8	8 to 16	.25 to 1	1 to .25
2.	15 to 5	5 to 15	.50 to 1	1 to .50
3.	3 to 4	4 to 3	.37½ to 1	1 to .37½
4.	¼ to ¾	¾ to ¼	.33⅓ to 1	1 to .33⅓
5.	⅖ to ⅘	⅘ to ⅖	.62½ to 1	1 to .62½
6.	1 ft. to 1 in.	1 in. to 1 ft.	1 pt. to 1 qt.	1 qt. to 1 pt.
7.	1 qt. to 1 pt.	1 pt. to 1 qt.	\$5 : \$15	\$15 to \$5

8. John has 8 cents and James has 4 cents. What is the ratio of John's money to James's money?

The question, "What is the ratio of John's money to James's money?" means the same as the question, "John's money is how many times James's money?" *Ans.* 8 : 4, or ⅘, or 2.

9. A can do a piece of work in 3 days. B can do the same piece of work in 5 days. What is the ratio of the time it takes A to the time it takes B?

Ans. $\frac{3}{5}$. The ratio expressed by a fraction is always abstract. It tells how many times or what part of a time the antecedent contains the consequent. The result is the quotient, or the ratio.

10. In problem 9, what is the ratio of the time it takes B to the time it takes A? How else may the question be asked?

11. In one bin there are 40 bushels of wheat; in a second bin there are 20 bushels of wheat. What is the ratio of the quantity of wheat in the first bin to the quantity in the second bin?

12. If a third bin contains 30 bushels, how many bushels must the fourth bin contain in order that the ratio of the quantity of wheat in the third bin to the quantity in the fourth bin shall be the same as the ratio of the quantity in the first bin to the quantity in the second bin?

13. Can you find the ratio of \$3 to 6 days?

NOTE. There can be no ratio between dollars and days, because dollars cannot be measured by days. The unit of the first number is \$1; the unit of the second number is 1 day. *Like numbers* have the same unit; as, \$3 and \$6; 4 days and 6 days; 8 hours and 4 hours. A ratio can exist only between two numbers having the *same unit*.

14. What is the ratio of 12 days to 3 days?

15. What is the ratio of 18 to 2?

16. What is the ratio of 4 to 4? What is the ratio of any number to itself? $4 : 4 = 7 : 7 = 8 : 8$, etc.

17. Is the ratio of a number to a larger number greater or less than 1? Illustrate.

Is the ratio of a number to a smaller number greater or less than 1? Illustrate.

Two ratios cannot be equal when one of them is greater than 1, and the other less than 1, or equal to 1.

WRITTEN

18. What is the ratio of 18 to 24?

19. What is the ratio of 25 to 6?

Express the following ratios as fractions, and tell which are greater than 1 and which are less than 1:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
20.	3 : 4	8 : 5	12 : 35	13 : 14	17 : 24	15 : 48
21.	25 : 55	24 : 32	16 : 50	18 : 40	15 : 65	65 : 15
22.	$\frac{2}{3} : \frac{4}{5}$	$\frac{1}{2} : \frac{1}{4}$	$2\frac{1}{2} : 1\frac{2}{3}$	$\frac{3}{5} : 2\frac{3}{8}$	$\frac{3}{4} : 2\frac{1}{8}$	$\frac{5}{6} : 1\frac{4}{5}$
23.	$3\frac{1}{3} : 6\frac{2}{3}$	$4\frac{1}{2} : 9$	$8 : 2\frac{2}{3}$	$3 : 4\frac{1}{2}$	$2\frac{1}{2} : 3\frac{1}{2}$	$5\frac{1}{3} : 8$

Find the value of the following ratios:

24. 2 yd. 1 ft. : 3 yd. 1 ft.

25. 10 gal. 1 qt. : 5 gal. 1 pt.

26. 1 lb. 1 oz. : 3 lb. 3 oz.

27. 4 pk. 1 qt. : 1 pk. $\frac{1}{2}$ pt.

SIMPLE PROPORTION

STUDY RECITATION

A **proportion** is an expression of equality between ratios. It may be written in three ways:

(*a*) 12 : 6 = 8 : 4, read, the ratio of 12 to 6 equals the ratio of 8 to 4.

(*b*) 12 : 6 :: 8 : 4, read, 12 is to 6 as 8 is to 4.

(*c*) $\frac{12}{6} = \frac{8}{4}$, read, twelve sixths equals eight fourths.

The first and fourth terms of a proportion are the **extremes**; the second and third terms are the **means**.

The first and third terms of a proportion are the **antecedents**; the second and fourth terms are the **consequents**.

The fourth term of a proportion is a **fourth proportional** to the other three terms.

When the second and third terms of a proportion are the same number, this number is a **mean proportional** to the other two. In $4 : 8 = 8 : 16$, 8 is a mean proportional to 4 and 16.

In every proportion, the product of the means equals the product of the extremes.

This is the principle upon which the solution of problems in proportion depends.

If three terms of a proportion are given, either two extremes and a mean, or two means and an extreme are given.

The product of the extremes divided by the given mean equals the other mean.

The product of the means divided by the given extreme equals the other extreme.

Study Recitation

Find the missing term in each of the following proportions:

1.	2.	3.	4.
$12 : 5 = 24 : ?$	$12 : 5 = ? : 10$	$12 : ? = 24 : 10$	$? : 5 = 24 : 10$

Solution

$\frac{\overset{2}{\cancel{5} \times \cancel{24}}}{\underset{}{\cancel{12}}} = 10$, *Ans.* $\quad \frac{\overset{2}{\cancel{12} \times \cancel{10}}}{\underset{}{\cancel{5}}} = 24$, *Ans.* $\quad \frac{\overset{5}{\cancel{12} \times \cancel{10}}}{\underset{2}{\cancel{24}}} = 5$, *Ans.* $\quad \frac{\overset{12}{\cancel{5} \times \cancel{24}}}{\underset{2}{\cancel{10}}} = 12$, *Ans.*

Solve:

5. $3 : 6 = 8 : ?$	9. $5 : ? = 6 : 12.$	13. $? : 9 = 8 : 18.$
6. $3 : 8 = ? : 16.$	10. $? : 5 = 10 : 10.$	14. $14 : ? = 7 : 10.$
7. $4 : 7 = 8 : ?$	11. $4 : ? = 8 : 10.$	15. $3 : 9 = ? : 15.$
8. $2 : 5 = ? : 20.$	12. $3 : 8 = 6 : ?$	16. $3 : ? = 4 : 16.$

PROBLEMS

WRITTEN

1. If a man earns $12 in 6 days, how many dollars can he earn in 28 days?

SOLUTION. As the answer is to be dollars, make 12 the third term. If a man earns $12 in 6 days, can he earn a greater or a less number of dollars in 28 days? *Ans.* A greater number. Therefore 28, the larger of the two numbers representing days, is the second term of the proportion and 6 is the first term.

The proportion is $6 : 28 = 12 : ?$ $\qquad \frac{28 \times \overset{2}{\cancel{12}}}{\cancel{6}} = 56.$ *Ans.* $56.

Most problems in proportion can be more easily solved by the fractional method or by analysis.

Fractions. 28 days is $\frac{28}{6}$, or $4\frac{2}{3}$, of 6 days. Therefore the man will earn $\frac{14}{\cancel{3}}$ of $\overset{4}{\cancel{12}}$, or $56.

Analysis. If a man earns $12 in 6 days, he earns $\frac{1}{6}$ of $12, or $2, in 1 day. He earns $28 \times \$2$, or $56, in 28 days.

TO THE TEACHER. The teacher who wishes to teach at this time "The Use of Symbols," pp. 387 to 399, can make use of this method as well.

2. If the interest on a note for 7 months is $35, what is the interest on the same note for 15 months?

(*a*) *Proportion.* $7 : 15 = 35 : ?$ $\qquad 15 \times \frac{\overset{5}{\cancel{35}}}{\cancel{7}} = 75.$ *Ans.* $75.

(*b*) *Fractions.* $\frac{15}{\cancel{7}}$ of $\overset{5}{\cancel{35}} = \$75.$

(*c*) *Analysis.* Interest for 1 month = $5.
Interest for 15 months = $15 \times \$5$, or $75.

3. How many days will it take 24 men to do a piece of work that can be done by 9 men in 4 days?

(*a*) *Proportion.* $24 : 9 = 4 : ?$ $\qquad \frac{\overset{3}{\cancel{4}} \times \cancel{9}}{\underset{2}{\cancel{24}}} = 1\frac{1}{2}.$ *Ans.* $1\frac{1}{2}$ da.

NOTE. 24 men take *less* time than 9 men. As the answer is to be less than 4 da., make the smaller number of men (9) the second term.

(*b*) *Fractions.* $\frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{24}}}$ of 4 da. $= \frac{3}{2}$ da. $= 1\frac{1}{2}$ da.

(*c*) *Analysis.* If 9 men take 4 da., 1 man takes 9×4 da., or 36 da., and 24 men take $\frac{1}{24}$ of 36 da., or $1\frac{1}{2}$ da.

4. How many bushels of wheat can be bought for \$60, if 12 bushels can be bought for \$6?

(*a*) *Proportion.* \$6 : \$60 = 12 : ? $\frac{12 \times \overset{10}{\cancel{60}}}{\cancel{6}} = 120$. *Ans.* 120 bu.

(*b*) *Fractions.* $\frac{\overset{10}{\cancel{60}}}{\cancel{6}}$ of 12 bu. = 120 bu.

(*c*) *Analysis with 1 bu. as unit.* If 12 bu. cost \$6, 1 bu. costs $\frac{1}{12}$ of \$6, or \$.50. For \$60 as many bushels can be bought as \$.50 is contained times in \$60, or 120 bu.

(*d*) *Analysis with \$1 as unit.* If for \$6, 12 bu. can be bought, for \$1, $\frac{1}{6}$ of 12 bu., or 2 bu., can be bought. For \$60, 60×2 bu., or 120 bu. can be bought.

Solve the following examples by the method that is easiest for you:

5. How many bushels of wheat can be bought for \$72, if 6 bushels can be bought for \$3?

6. Thirty men can do a piece of work in 6 days. At the same rate, how long will it take 18 men to do the same work?

7. If 5 men can do a piece of work in 20 days, how many men can do the work in $12\frac{1}{2}$ days?

8. A piece of lead pipe $5\frac{1}{2}$ ft. long weighs 11 pounds. What is the weight of a piece 17 ft. in length?

9. How much wheat can be bought for $150, if 20 bushels can be bought for $16?

10. A contractor requires 80 days to build a house when his men work 10 hours a day. How many days will he require when his men work 8 hours a day?

11. A freight train runs 20 miles an hour, and a passenger train on the same road runs 35 miles an hour. If the passenger train makes a trip between two points in 4 hours, how many hours will it require for the freight train to make the same trip?

12. The rates of two steamboats are as 9 to 12. If the faster boat makes a trip in 6 hours, how long will it take the slower boat to make the same trip?

13. A contractor agrees to do a piece of work in 40 days, and employs 20 men upon it. The work is half finished at the end of 30 days. How many additional workmen must he employ in order to finish the work on time?

SUGGESTION. Before beginning the solution, change the statement of the problem: At the end of 30 days as much work remains to be done as has already been done.

The problem may then be stated: If 20 men do a piece of work in 30 days, how many men will it require to do an equal piece of work in 10 days?

14. If a loaf of bread costs 4¢ when flour is $6 a barrel, how much should it cost when flour is $7½ a barrel?

15. If 4 men or 6 boys can do a piece of work in 12 days, in how many days will 6 men and 3 boys do the same piece of work?

16. If 4 farm hands can build a fence in 3 days, how many farm hands would it take to build a similar fence in ½ a day?

PROPORTIONAL PARTS

1. Divide the number 12 into two parts in the ratio of 2 to 4.

SOLUTION. For every 2 units in the smaller part, there are 4 units in the larger part or 6 units in both parts. The smaller part is $\frac{2}{6}$ of the whole, or $\frac{2}{6}$ of 12, or 4. The larger part is $\frac{4}{6}$ of the whole, or $\frac{4}{6}$ of 12, or 8.

Test. $4 + 8 = 12$. $4 : 8 = 2 : 4$.

2. Divide the number 24 into two parts whose ratio equals the ratio of 3 to 5.

3. Separate 36 into two parts whose ratio equals the ratio of 5 to 7.

4. Separate 15 into two parts whose ratio equals the ratio of $\frac{1}{2}$ to $\frac{1}{3}$.

SUGGESTION. Before beginning the analysis, reduce $\frac{1}{2}$ and $\frac{1}{3}$ to 6ths. The ratio then becomes $\frac{3}{6} : \frac{2}{6}$, which is the same ratio as 3 : 2, and the requirement is to separate 15 into two parts whose ratio equals the ratio of 3 to 2.

5. 20 pieces of stock for woodwork are divided between James and John so that the number of pieces James has is to the number John has as 2 is to 3. How many has each?

6. If John earns $\frac{3}{4}$ as much per day as Henry, what is the ratio of John's earnings to Henry's earnings? What is the ratio of Henry's earnings to John's earnings?

7. Divide 40 pieces of stock between James and John so that the number James has is $\frac{3}{5}$ the number John has.

8. Two men are 30 miles apart and are traveling toward each other. One travels 2 miles an hour, and the other 3 miles an hour. How far will each have traveled when they meet? In how many hours will they meet?

9. A garden 18 ft. long is divided into 3 beds whose lengths are to each other as 2, 3, and 4. What is the length of each bed?

PERCENTAGE

Study Recitation

1. What is $\frac{1}{2}$ of 24? $\frac{1}{4}$ of 48? $\frac{2}{5}$ of 35? $\frac{3}{4}$ of 32?

Finding each result is finding a fractional part of a number.

2. What is $\frac{50}{100}$ of 24? $\frac{25}{100}$ of 48? $\frac{40}{100}$ of 35? $\frac{75}{100}$ of 32?

Finding each result is finding a number of hundredths of a number. Notice that the fractions in example 1 are reduced to hundredths in each case in example 2.

3. What is .5 of 24? .25 of 48? .4 of 35? .75 of 32?

Finding each result is finding a decimal part of a number. Notice that the fractions in example 1 are changed to decimals in example 3, and that these decimals equal the fractions in example 2. Thus, $.5 = \frac{5}{10}$; $.25 = \frac{25}{100}$, etc.

Per cent means **per hundred** or **hundredths.** The sign for it is %.

Percentage is the part of arithmetic that treats of per cent.

$\frac{1}{2} = \frac{50}{100} = .50 = 50\ \%$
$\frac{1}{4} = \frac{25}{100} = .25 = 25\ \%$
$\frac{2}{5} = \frac{40}{100} = .40 = 40\ \%$
$\frac{3}{4} = \frac{75}{100} = .75 = 75\ \%$
$7\ \%$ means .07, or $\frac{7}{100}$
$30\ \%$ means .30, or $\frac{30}{100}$, or $\frac{3}{10}$
$12\frac{1}{2}\ \%$ means $.12\frac{1}{2}$, or .125, or $\frac{12\frac{1}{2}}{100}$, or $\frac{1}{8}$

Write as per cent:

4. .04 .06 .07 .10 .25 .48 .96 .68 .33

Write as decimals:

5. $2\ \%$ $5\ \%$ $25\ \%$ $30\ \%$ $75\ \%$ $87\frac{1}{2}\ \%$ $33\frac{1}{3}\ \%$ $12\frac{1}{2}\ \%$

Note. $87\frac{1}{2}\% = .87\frac{1}{2} = .875$; $33\frac{1}{3}\% = .33\frac{1}{3}$.

Learn the following table:

$\frac{1}{2} = 50\,\%$	$\frac{2}{5} = 40\,\%$	$\frac{3}{8} = 37\frac{1}{2}\,\%$	$\frac{1}{20} = 5\,\%$
$\frac{1}{3} = 33\frac{1}{3}\,\%$	$\frac{3}{5} = 60\,\%$	$\frac{5}{8} = 62\frac{1}{2}\,\%$	$\frac{1}{25} = 4\,\%$
$\frac{2}{3} = 66\frac{2}{3}\,\%$	$\frac{4}{5} = 80\,\%$	$\frac{7}{8} = 87\frac{1}{2}\,\%$	$\frac{5}{4} = 125\,\%$
$\frac{1}{4} = 25\,\%$	$\frac{1}{6} = 16\frac{2}{3}\,\%$	$\frac{1}{10} = 10\,\%$	$\frac{9}{8} = 112\frac{1}{2}\,\%$
$\frac{3}{4} = 75\,\%$	$\frac{5}{6} = 83\frac{1}{3}\,\%$	$\frac{1}{12} = 8\frac{1}{3}\,\%$	$\frac{4}{3} = 133\frac{1}{3}\,\%$
$\frac{1}{5} = 20\,\%$	$\frac{1}{8} = 12\frac{1}{2}\,\%$	$\frac{1}{16} = 6\frac{1}{4}\,\%$	$\frac{5}{3} = 166\frac{2}{3}\,\%$

NOTE. $\frac{1}{2}\,\%$ means $\frac{1}{2}$ of 1%, or $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$; $\frac{1}{8}\,\%$ means $\frac{1}{8}$ of 1%, or $\frac{1}{800}$.

Tell what per cents correspond to the fractions given below, and what fractions in their lowest terms correspond to the per cents and decimals given below:

6. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{3}{10}$, $\frac{1}{10}$, $\frac{1}{8}$.

7. $\frac{8}{20}$, $\frac{7}{20}$, $\frac{9}{20}$, $\frac{11}{20}$, $\frac{3}{25}$, $\frac{6}{25}$, $\frac{9}{25}$, $\frac{8}{25}$, $\frac{11}{25}$, $\frac{12}{25}$.

8. 25%, 10%, 20%, 75%, 80%, 60%, 40%, 30%, 50%, 70%.

9. .75, .20, .50, .60, .10, .25, .80, .70, .30, .40, .90, .35.

To express per cent as a common fraction, write the number of per cent as the numerator and 100 *as the denominator. Reduce this fraction to its lowest terms.*

Thus, $25\,\% = \frac{25}{100} = \frac{1}{4}$. $\quad 37\frac{1}{2}\,\% = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}$.

To express per cent as a decimal, remove the sign % and place a decimal point two places to the left.

Thus, $25\,\% = .25$; $\quad 125\,\% = 1.25$; $\quad 2\frac{1}{2}\,\% = .02\frac{1}{2} = .025$; $15.5\,\% = .155$.

To express a common fraction as per cent, change it to hundredths and use the sign % instead of the denominator 100.

Thus, $\frac{1}{4} = \frac{25}{100} = 25\,\%$. $\quad \frac{3}{8} = \frac{37\frac{1}{2}}{100} = 37\frac{1}{2}\,\%$.

To express a decimal as per cent, omit the decimal point and write the sign %.

Thus, $.25 = 25\%$; $.33\frac{1}{3} = 33\frac{1}{3}\%$; $1.75 = 175\%$; $1.87\frac{1}{2} = 187\frac{1}{2}\%$; $1.625 = 162\frac{1}{2}\%$.

Express as common fractions, integers, or mixed numbers:

	a.	*b.*	*c.*	*d.*	*e.*
10.	2%	$33\frac{1}{3}\%$	125%	$112\frac{1}{2}\%$	$\frac{1}{2}\%$
11.	5%	$66\frac{2}{3}\%$	120%	$137\frac{1}{2}\%$	$\frac{3}{4}\%$
12.	7%	$6\frac{1}{4}\%$	100%	$162\frac{1}{2}\%$	$\frac{1}{5}\%$
13.	10%	$8\frac{1}{3}\%$	200%	$166\frac{2}{3}\%$	$\frac{5}{8}\%$
14.	12%	$12\frac{1}{2}\%$	175%	$187\frac{1}{2}\%$	$\frac{1}{10}\%$
15.	15%	$16\frac{2}{3}\%$	105%	$233\frac{1}{3}\%$	$.5\%$
16.	20%	$83\frac{1}{3}\%$	150%	$116\frac{2}{3}\%$	$.05\%$
17.	25%	$37\frac{1}{2}\%$	165%	$133\frac{1}{3}\%$	$.005\%$
18.	29%	$87\frac{1}{2}\%$	300%	$108\frac{1}{3}\%$	$.25\%$
19.	34%	$62\frac{1}{2}\%$	275%	$162\frac{1}{2}\%$	$.025\%$

Express the above per cents as decimals.

Express first as per cents; then as decimals:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
20.	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{7}{25}$	$\frac{7}{4}$	$3\frac{1}{8}$	20
21.	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{9}{8}$	$4\frac{1}{4}$	30
22.	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{11}{50}$	$\frac{4}{3}$	$2\frac{1}{5}$	25
23.	$\frac{1}{16}$	$\frac{9}{10}$	$\frac{1}{40}$	$\frac{7}{3}$	$5\frac{1}{3}$	$33\frac{1}{3}$
24.	$\frac{5}{16}$	$\frac{8}{20}$	$\frac{3}{50}$	$\frac{6}{5}$	$3\frac{3}{4}$	$12\frac{1}{2}$
25.	$\frac{1}{3}$	$\frac{7}{50}$	$\frac{5}{12}$	$\frac{17}{16}$	$4\frac{7}{8}$	$37\frac{1}{2}$

Express as per cents:

26.	.25	1.25	2.50	$1.12\frac{1}{2}$	$1.62\frac{1}{2}$
27.	$.37\frac{1}{2}$	1.20	3.75	$2.37\frac{1}{2}$	$1.66\frac{2}{3}$
28.	$.03\frac{3}{4}$	1.50	4.80	$3.62\frac{1}{2}$	$1.87\frac{1}{2}$
29.	$.33\frac{1}{3}$	1.60	5.90	$1.16\frac{2}{3}$	$2.06\frac{2}{3}$
30.	$.04\frac{1}{2}$	1.75	6.40	$1.33\frac{1}{3}$	$1.05\frac{1}{4}$

31. Find $\frac{1}{2}$ of each of the following numbers:

12, 18, 25, 30, 62, 48, 256, 312.

32. Find 50% of each of the above numbers by multiplying them by .50.

33. Find 25% of each of the following numbers by multiplying them by .25.

16, 24, 60, 180, 254, 82, 96, 480, 320, 1600, 5600, 3840.

34. Prove your work by finding $\frac{1}{4}$ of each of these numbers.

35. 7% of a number equals what part of it? what decimal of it? Find $\frac{7}{100}$ of 365. Find .07 of 365 by using the decimal. Which is the better way?

36. Find 25% of 36 by using the per cent as a fraction in its lowest terms. Find 25% of 36 by multiplying by the decimal .25. Which is the better way and why?

To find any per cent of a number.

STUDY RECITATION

1. Find 10% of 30, or $\frac{1}{10}$ of 30.

SOLUTION. $10\% = \frac{1}{10}$; $\frac{1}{10}$ of $30 = 3$.

Every example in which it is required to *find a per cent of a number* may be changed to an example in common fractions, or in decimals.

2. Find $37\frac{1}{2}\%$ of 240.

$$\begin{array}{r} 240 \\ .37\frac{1}{2} \\ \hline 80 \\ 16\ 80 \\ 72\ 0 \\ \hline 89.60 \end{array}$$

3. Find $37\frac{1}{2}\%$ of 240.

$37\frac{1}{2}\% = \frac{3}{8}$; $\frac{3}{8}$ of $240 = 90$.

In example 2, change $37\frac{1}{2}\%$ to a decimal and multiply.

In example 3, change $37\frac{1}{2}\%$ to $\frac{3}{8}$; $\frac{3}{8}$ of 240 may be found mentally.

4. Find $\frac{3}{4}\%$ of \$480. 5. Find .05% of \$6244.

\$480	In example 4, change $\frac{3}{4}\%$ to $.00\frac{3}{4}$; then multiply.	\$6244
$.00\frac{3}{4}$	In example 5, change .05% to .0005; then multiply.	.0005
\$3.60		\$3.1220

Reduce the given per cent to a fraction in its lowest terms and find this fractional part of the number. Or,

Multiply the number by the given per cent expressed as a decimal.

Change the following examples in percentage to examples in common fractions, and to examples in multiplication of decimals. Fill the blanks, and solve:

	Percentage	Common Fractions	Multiplication of Decimals
6.	Find 75% of 48.	Find $\frac{3}{4}$ of 48.	Find .75 of 48.
7.	Find $66\frac{2}{3}\%$ of 21.	Find —— of 21.	Find —— of 21.
8.	Find 10% of 40.	Find —— of 40.	Find —— of 40.
9.	Find 10% of 25.	Find —— of 25.	Find —— of 25.
10.	Find 25% of 18.	Find —— of 18.	Find —— of 18.
11.	Find 40% of 30.	Find —— of 30.	Find —— of 30.

Solve, changing the given per cents to common fractions:

12. 5% of 40; 60; 80; 100; 140; 120; 240; 600; 500.
13. 4% of 50; 100; 200; 75; 125; 150; 600; 700; 800.
14. $12\frac{1}{2}\%$ of 24; 32; 40; 64; 80; 56; 48; 72; 88; 96; 144.
15. $33\frac{1}{3}\%$ of 18; 9; 12; 15; 39; 45; 27; 30; 36; 42; 51.
16. $66\frac{2}{3}\%$ of 21; 12; 15; 30; 24; 27; 18; 33; 36; 42; 54.
17. 20% of 25; 10; 15; 55; 35; 50; 20; 30; 40; 45; 125.
18. 75% of 24; 8; 12; 16; 28; 36; 40; 44; 48; 100.
19. $37\frac{1}{2}\%$ of 32; 40; 56; 64; 80; 72; 48; 88; 96; 24; 104.

Written

Solve, using the given per cents as decimals:

20. 7% of 28; 39; 251; 175; 600; 500; 400; 300; 1000.

21. 12% of 36; 49; 87; 120; 140; 160; 180; 240; 300.

22. 6% of 18; 36; 45.6; 27.3; 5.5; 4.6; 7.2; 900; 500.

23. 8% of 2.7; 3.5; 2.78; 15.6; 3.4; 5.5; 6.5; 750; 800.

24. 3% of 150; 48; 273⅓; 28⅔; 5.4; 6.5; 7.8; 940; 870.

25. 3½% of 48; 24; 36; 125; 27; 3.5; 6.7; 5.4; 800; 700.

26. 7½% of 25; 40; 20; 35; 426; 800; 640; 528; 416.

27. 15% of $80.

28. 8⅓% of $36.

29. 37½% of 48 lb.

30. 50% of 12 oz.

31. 12½% of 84 da.

32. 60% of $18.

33. 75% of 12 mo.

34. ½% of $400.

First find 1% of $400, and then ½%.

35. 100% of 12 eggs.

36. ⅔% of $1200.

37. 200% of 6 cents.

38. 6¼% of 64.

39. 1½% of $100.

40. 2½% of 200.

41. 90% of 81.

42. 5/7% of 91.

43. 4% of $250.

44. 10% of 87.50 acres.

45. 150% of 12 eggs.

46. 137½% of $800.

47. 133⅓% of $25.

Work by both rules, and decide which one is the better:

48. 12½% of 24; 56; 83; 128; 250; 300; 600; 900.

49. 25% of 16; 28; 34; 46; 142; 200; 300; 400.

50. 120% of 15; 35; 36; 48; 27; 100; 150; 300.

51. 175% of .28; 18; 19; 42; 126; 400; 800; 900.

Find first 25 %; then 50 %; then $37\frac{1}{2}$ %; then $112\frac{1}{2}$ %; then 5 % of:

	a.	*b.*	*c.*	*d.*	*e.*
52.	$200	15620	154 mi.	$400.80	16000
53.	$316	17890	264 yd.	$316.40	18000
54.	$2480	44000	344 gal.	$9480.48	24000

Find first 2 %; then $3\frac{1}{2}$ %; then $5\frac{3}{4}$ %; then 6 % of:

55.	1640	15780	$125.50	$2340.90	$7080.25
56.	2970	20000	$116.80	$3500.40	$2040.50
57.	8341	50000	$240.54	$6000.70	$9854.75

Find first $37\frac{1}{2}$ %; then $137\frac{1}{2}$ %; then $133\frac{1}{3}$ %; then $\frac{1}{4}$ %; then .05 % of:

58.	2030	18500	$640.50	$6500.50	$10000
59.	3050	17800	$730.25	$9250.75	$25000
60.	6070	19400	$825.16	$8045.60	$80000

PROBLEMS

Oral

1. From a tank containing 50 gal. of vinegar, 10 % leaked out. How many gallons were lost? How many were left in the cask?

Solution. 10 % of a number is $\frac{1}{10}$ of it; $\frac{1}{10}$ of 50 gal. is 5 gal., which leaked out. There were left 50 gal. less 5 gal., or 45 gal.

2. In an orchard there are 85 trees. If 20 % of them are apple trees, and the others are pear trees, how many trees of each kind are there in the orchard?

3. A man who has an income of $ 6770 a year pays an income tax of 1 % on the part of his income that exceeds $ 4000. What is the tax?

4. Find the income tax of a woman who pays 1 % on her income in excess of $3000, if she has a salary of $2000, receives $1000 a year rent, and $500 interest on bonds.

5. A merchant bought cloth at 80 ¢ a yard, and sold it for 25 % more than it cost. For how much more than the cost did he sell it? What was the selling price per yard?

6. A piece of cloth that cost $1.25 a yard was sold at a loss of 20 %. How much less than the cost was received for it? What was the selling price?

7. If a clothing dealer pays $15 apiece for overcoats, and sells them at a gain of 20 %, what is the selling price? How much does he make on one coat? on 1 dozen coats?

8. A farmer tests 80 kernels of corn to see how many will grow. $12\frac{1}{2}$ % fail to grow. How many kernels grow? How many fail to grow?

9. There are 40 words in a spelling lesson. If a boy misses 10 % of them, how many does he spell correctly?

10. If I fail to work 5 % of the problems in my arithmetic lesson, and there are 20 problems in the lesson, how many do I work? What per cent of them do I work? How many do I fail to work?

11. A farmer harvested 250 bu. of potatoes, but 10 % of them were spoiled by freezing. How many bushels were spoiled? How many bushels were not damaged?

12. If a man buys a horse for $88, and sells it so as to gain $12\frac{1}{2}$ %, for how much does he sell the horse? How much does he gain?

13. A man sold $\frac{1}{3}$ of his farm. What per cent of it did he sell? How many hundredths of it did he sell? How many hundredths of the farm had he left? What per cent of the farm had he left?

14. A pupil solved correctly $\frac{3}{4}$ of the problems in his lesson. What part of the problems in his lesson had he still to solve? What per cent of the problems did he solve correctly? What per cent of the problems had he still to solve?

15. A man has 30 tons of coal and sells 10 % of it at $4 a ton. How much does he receive for what he sells? How much is the rest of the coal worth at the same rate?

16. About $16\frac{2}{3}$ % of a pound of butter is salt and water. How many pounds of butter can be made from 190 lb. of butter fat?

HINT. $16\frac{2}{3}\% = \frac{1}{6}$. $\frac{5}{6}$ of 1 lb. of butter is butter fat. 190 lb. $\div \frac{5}{6} =$ 228 lb. *Ans.*

17. A cow whose milk tested 4 % gave 4800 lb. of milk a year. How many pounds of fat did the cow produce?

NOTE. A test of 4 % means 4 % of butter fat.

18. If $\frac{1}{4}$ of a box of lemons is sold, and $\frac{1}{8}$ of the remainder is unsalable, what per cent of the lemons is not salable?

WRITTEN

19. A real estate dealer bought 1275 acres of land, and sold $66\frac{2}{3}$ % of it to one man, and 20 % of the remainder to another man. How many acres did each purchase, and how many remained?

20. If bread made from a sack of flour weighs $33\frac{1}{3}$ % more than the flour, what is the weight of the bread made from a sack of flour weighing 50 lb.?

21. Under the 1913 Federal Reserve Act, country banks must hold 12 % of their demand deposits in reserve. If the demand deposits of the Farmers' Bank amount to $185,465.17, how much must be reserved?

Farm Problems

22. A cow gave 22 lb. of milk at night and 19 lb. in the morning. The test showed the evening's milk to contain 4.2 % of butter fat and the morning's milk 4.3 %. How much butter fat did the cow produce at the two milkings?

23. Mr. A took 62 lb. of cream testing 28 % to the creamery. What was its value at 35 cents per pound for butter fat?

24. A farmer delivered the following amounts of cream to a creamery: Monday, 196 lb.; Tuesday, 92 lb.; Wednesday, 99 lb.; Thursday, 102 lb.; Friday, 93 lb.; and Saturday, 95 lb. The average test for the week was 32.5 %, and butter fat was worth $.35 a pound. What was the amount of his cream check for the week? If this was the income from 25 cows, what was the average per cow?

25. Corn silage contains the following digestible food materials: protein, 1.2 %; carbohydrates, 14 %; and fat, 0.7 %. How many pounds of each are there in a ton (2000 pounds) of silage?

26. There is no perfect method for the separation of fat from milk, so there is some left in skim milk. The following are the amounts under varying conditions:

Shallow setting	0.8 per cent
Deep setting	0.2 per cent
Separator	0.05 per cent

What would be the loss with 1000 lb. of milk under each system?

27. A farmer has a herd of 20 cows that average 6000 lb. of milk per year. What would be the loss in pounds of butter fat per year under each system? Find the value of the butter fat lost in each case if the average price is $.30 per pound.

28. A farmer tested the skim milk from his separator and found it to contain .13 % of butter fat. The separator was adjusted, and a second test showed .05 % of butter fat. If the separator had been out of adjustment 30 days, how great was his loss with a herd of 20 cows averaging 25 pounds of milk per day, butter fat being valued at 28 cents per pound?

29. The average composition of butter is:

Fat	83 per cent
Water	13 per cent
Casein	1 per cent
Salt	3 per cent

The output of a creamery for one week was 50 tubs, of 60 lb. each, of butter having the average composition. How much was fat? water? casein? salt?

30. According to the reports of the American Guernsey Cattle Club, a Guernsey produced during one season 11,457 lb. of milk testing 4.97 %. Find the total pounds of butter fat produced.

31. The average production of 326 cows in the Lyndeboro (Mass.) Cow Test Association for a period of one year was 6463.2 lb. of milk testing 3.76 % fat. What was the average production of butter fat? What was the total for all herds?

The following is the record of a cow by periods, each four weeks in length. Determine for each period the number of pounds of butter fat produced and the profit above the cost of feed:

	Periods	Pounds of Milk	Test	Value Butter	Cost of Stall Feed
32.	1	650.4	3.8 %	$8.69	$5.32
33.	2	631.2	3.8	8.10	5.32
34.	3	631.1	3.9	8.61	3.78
35.	4	705.7	3.5	8.61	4.97

	Periods	Pounds of Milk	Test	Value Butter	Cost of Stall Feed
36.	5	646.4	4.0%	$7.50	$3.99
37.	6	615.7	4.0	7.17	Pasture
38.	7	490.9	4.2	6.00	Pasture
39.	8	275.0	4.2	3.62	Pasture
40.	9	55.8	4.2	.81	Pasture
41.	10	Dry			
42.	11	826.7	3.8	11.35	3.77
43.	12	867.3	3.7	13.06	5.85
44.	13	851.7	4.0	15.11	6.20

In the following, how many pounds of butter fat were produced by all the mature cows? by the four-year-olds? the three-year-olds? the two-year-olds?

	Pounds Milk	Per Cent	Pounds Fat
45. 389 mature cows average	10492	3.83	?
46. 131 four-year-olds average	9487	3.95	?
47. 178 three-year-olds average	8981	3.94	?
48. 300 two-year-olds average	7761	4.03	?

49. One cow in a herd gave in the morning 25 lb. of milk testing 4 % and in the evening 30 lb. testing 3.8 %. What was the average per cent of butter fat that day? the total weight of the butter fat?

50. Another cow in the same herd gave a total of 65 lb. of milk that day, with an average test of 3.1 %. Which produced the greater amount of butter fat?

51. At $.25 a pound, find the amount received during May for butter fat from the cows in examples 49 and 50.

Find the number of pounds of fat in each of the following:

52. 50 lb. of milk, 5.0 % fat.

53. 40 lb. of milk, 4.8 % fat.

54. 25 lb. of milk, 4.5 % fat.

55. 80 lb. of milk, 4.0 % fat.

56. 75 lb. of milk, 4.4% fat. **57.** 45 lb. of milk. 3.5% fat.

58. Separator cream usually contains from 15% to 35% fat. Which is the more profitable to sell, 1000 lb. of cream at $.15 a pound, or butter fat at $.25 a pound, if the cream contains 25% butter fat?

The following are approximate percentages of plant-food constituents in fresh manure from various farm animals. How many pounds of nitrogen, phosphoric acid, and potash are there in one ton of each manure, and what is the value of 1 ton of each, if nitrogen is worth $.15 per pound and phosphoric acid and potash are each worth $.05 per pound?

	Per Cent of Nitrogen	Per Cent of Phosphoric Acid	Per Cent of Potash
59. Horse	0.58	0.28	0.53
60. Cow	0.44	0.16	0.40
61. Pig	0.45	0.19	0.60
62. Sheep	0.83	0.23	0.67
63. Poultry	1.60	1.50	0.80

64. The following has been suggested as a good ration for a dairy cow weighing 1000 lb. and giving 15 lb. of milk per day: 10 lb. of a mixture of one third each of corn meal, ground oats and bran; 35 lb. of corn silage; 15 lb. of medium red clover. How many pounds of each would be required to feed a herd of 50 cows per day?

What would be the number of pounds of dry matter and the fertilizing constituents fed a herd of 50 cows per day, basing computations upon the following table:

	Per Cent of Dry Matter	Per Cent of Fertilizers		
		Nitrogen	Phosphoric Acid	Potash
65. Corn meal	87.1	1.58	.63	.40
66. Oats	88.9	1.86	.77	.59
67. Bran	88.3	2.67	2.89	1.61
68. Silage	22.0	.28	.11	.37
69. Clover	88.7	2.07	.38	2.20

70. If 80 % of the fertilizing constituents of the feed for the 50 cows in examples 65 to 69 are recovered in the manure, what would be the amount of nitrogen, phosphoric acid, and potash in the manure for a period of 365 days?

71. The surface 8 inches of Sheboygan County (Wisconsin) peat soil weighed about 350,000 pounds per acre. It contained: nitrogen 3.39 %, phosphoric acid .157 %, and potash .485 %. How many pounds of each were there in an acre of this surface soil?

To find what per cent one number is of another.

STUDY RECITATION

1. What *fractional part* of 10 is 8? What *decimal part?*

SOLUTION. 8 is $\frac{8}{10}$, or $\frac{4}{5}$, of 10; 8 is $\frac{8}{10}$, or .8, of 10.

What per cent of 10 is 8?

SOLUTION. 8 is $\frac{8}{10}$ of 10; $\frac{8}{10} = .80$, or 80 %.

2. 7 is what per cent of 14?

SOLUTION. 7 is $\frac{7}{14}$, or $\frac{1}{2}$, of 14. $\frac{1}{2} = 50\,\%$.

Notice that this is a special case of the problem *To find what fractional part one number is of another.* See p. 136.

What per cent of:

3.	25 is 5?	**6.**	\$48 is \$6?	**9.**	10 is 50?	**12.**	1 is $\frac{1}{2}$?
4.	60 is 2?	**7.**	\$54 is \$9?	**10.**	12 is 48?	**13.**	$\frac{3}{4}$ is $\frac{1}{4}$?
5.	72 is 9?	**8.**	\$42 is \$7?	**11.**	16 is 64?	**14.**	$\frac{4}{5}$ is $\frac{1}{5}$?

SOLUTION TO EX. 9. 50 is $\frac{50}{10}$, or $\frac{500}{100}$, or 500 %, of 10.

SOLUTION TO EX. 13. To find what per cent $\frac{1}{4}$ is of $\frac{3}{4}$, divide $\frac{1}{4}$ by $\frac{3}{4}$. $\frac{1}{4} \div \frac{3}{4} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3} = 33\frac{1}{3}\,\%$. Or, $\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3} = \frac{1}{3} = 33\frac{1}{3}\,\%$.

15. What per cent of $2460 is $123?

$$\begin{array}{r} .05 = 5\,\% \\ \$2460\overline{)\$123.00} \\ 123\ 00 \\ \hline \end{array}$$

$123 is $\frac{123}{2460}$ of $2460. Divide $123 by $2460. The result is .05, or 5%.

Use the number following "what per cent of" as divisor and the other number as dividend, and express the quotient as per cent.

WRITTEN

What per cent of:

16. 480 is 180?	**20.** $1600 is $600?	**24.** 500 is 600?
17. 1452 is 252?	**21.** $167 is $110.22?	**25.** 400 is 1200?
18. 2848 is 356?	**22.** $360 is $240?	**26.** 350 is 750?
19. 1989 is 221?	**23.** $185 is $111?	**27.** 600 is 720?

PROBLEMS

ORAL

1. What per cent of a quart is a pint?

2. What per cent of a yard is a foot?

3. A man earns $30 a month. His expenses for the year are $240. What per cent of his earnings does he save?

4. A boy earns $6 a week, and pays $4 a week for living expenses. What per cent of his weekly wages does he save?

5. If cloth is bought at 50¢ a yard and sold at 60¢ a yard, the gain is what per cent of the cost? The selling price is what per cent of the cost?

6. If a milkman sells milk at 7¢ a quart, which costs 5¢, what per cent of the cost does he gain?

7. In a field of 20 acres there are 15 acres of corn. What per cent of the field is corn?

8. From a pile of wood containing 24 cords, 20 cords are sold. What per cent of the wood is sold? What per cent remains unsold?

9. 8 hours is what per cent of a day? 15 seconds is what per cent of a minute?

10. 1 ounce is what per cent of a pound?

Written

The following is the equipment for four tables in a domestic science kitchen:

Iron ware	$2.00	Wooden ware	$ 2.50
Tin ware	3.50	Granite and enamel ware	14.00
Cutlery	5.50	Earthenware and glassware	6.50
Wire ware	3.00	Silverware	9.00
		Total	$46.00

What per cent of the total expense is for

11. Iron ware?
12. Tin ware?
13. Cutlery?
14. Wire ware?
15. Wooden ware?
16. Granite and enamel ware?
17. Earthenware and glassware?
18. Silverware?

Analyses of several brown silt loams found in Illinois showed the following weights of plant food constituents in 2,000,000 lb. of soil:

Soil	Nitrogen	Phosphoric Acid	Potash
19. 1	4370	1170	32240
20. 2	4840	1200	32940
21. 3	4290	1190	35340
22. 4	4910	1220	32960
23. 5	5050	1190	36250
24. 6	6750	1410	45020

Find the per cent of nitrogen, phosphoric acid, and potash in each soil.

25. A prize cow yielded, during one year, 27404.4 lb. of milk that contained 1058.34 lb. of butter fat. What was her test?

26. Another cow yielded, during one year, 26458.5 lb. of milk that contained 1058:34 lb. of butter fat. What was her test?

27. The record for Guernseys is held by Spottswood Daisy Pearl. In one year she gave 18602.75 lb. of milk which contained 957.38 lb. of butter fat. What was her average test?

28. Eminent's Bess holds the world's record for Jersey cows. During the year closing Sept. 11, 1913, she produced 18782.88 lb. of milk which contained 1132.75 lb. of butter fat. What was the average per cent of butter fat in her milk?

29–32. In examples 32–35 on p. 293, find the percentage of profit above the cost of feed for each period.

To find a number when a certain per cent of it is given.

STUDY RECITATION

1. 9 is $\frac{1}{4}$ of what number? 9 is .25 of what number? 9 is 25% of what number?

Notice that this is a special case of the problem *To find a number when a fractional part of it is given.* See p. 140.

2. 12 is $\frac{1}{5}$ of what number? 20% of what number?

3. 16 is $\frac{1}{2}$ of what number? 50% of what number?

4. 80 is $\frac{2}{5}$ of what number?

SOLUTION. If 80 is $\frac{4}{5}$ of a number, $\frac{1}{5}$ of it = $\frac{1}{4}$ of 80, or 40; $\frac{5}{5}$ of it = 5 × 40, or 200. Or, $80 \div \frac{4}{5} = \overset{40}{\cancel{80}} \times \frac{5}{\cancel{4}} = 200$.

5. 80 is 40 % of what number?

SOLUTION. 1 % of the number = $\frac{1}{40}$ of 80, or 2; 100 % of the number = 100 × 2, or 200. Or, 80 ÷ .40 = 200.

6. 14 is 3 % of what number?

SOLUTION. Substituting $\frac{3}{100}$ for 3 %, we have,

14 is $\frac{3}{100}$ of what number?

If 14 is $\frac{3}{100}$ of some number, $\frac{1}{100}$ of that number = $\frac{1}{3}$ of 14, or $\frac{14}{3}$, and $\frac{100}{100}$ of the number = 100 × $\frac{14}{3}$, or $\frac{1400}{3}$, or 466$\frac{2}{3}$.

ORAL

Change the following examples in percentage to examples in fractions. Reduce any number of hundredths to lowest terms:

PERCENTAGE	COMMON FRACTIONS
7. 8 is 25 % of what number?	Changed, becomes 8 is $\frac{1}{4}$ of what number?
8. 24 is 33$\frac{1}{3}$ % of what number?	Changed, becomes 24 is $\frac{1}{3}$ of what number?
9. 12 is 66$\frac{2}{3}$ % of what number?	Changed, becomes 12 is $\frac{2}{3}$ of what number?
10. 35 is 50 % of what number?	Changed, becomes _____
11. 16 is 80 % of what number?	Changed, becomes _____
12. 5$\frac{1}{2}$ is 25 % of what number?	Changed, becomes _____
13. 7 is 20 % of what number?	Changed, becomes _____
14. 9 is 75 % of what number?	Changed, becomes _____

Written

When a certain per cent of a number is given, the number may also be found without reducing the per cent to a fraction.

15. 21 is 7 % of what number?

$21 \div 7 = 3$. $100 \times 3 = 300$. Or,

$$\begin{array}{r} 3\ 00 \\ \text{¢}07.\overline{)21\text{¢}00.} \end{array}$$

If 21 is 7 % of a number, 1 % of the number is $\frac{1}{7}$ of 21, or 3. 100 % of the number is 100 times 3, or 300.

I. *To find a number when a certain per cent of it is given, divide the number expressing the part by the rate per cent used as a whole number, and multiply the quotient by* 100. *Or,*

II. *Divide the number expressing the part by the rate per cent used as a decimal.*

The second rule is more convenient when the number expressing the rate per cent will not exactly divide the number expressing the part.

16. 25 is 8 % of what number?

$$\begin{array}{r} 8\overline{)25} \\ 3\frac{1}{8} \end{array} \qquad \begin{array}{r} 100 \\ 3\frac{1}{8} \\ \hline 12\frac{1}{2} \\ 300 \\ \hline 312\frac{1}{2}, \text{ or} \\ 312.5 \end{array}$$

Or,

$$\begin{array}{r} 3\ 12.5 \\ \text{¢}08.\overline{)25\text{¢}00.0} \end{array}$$

17. 24 is 12 % of what number?

Analysis. If 24 is 12 % of a number, 1 % of the number is $\frac{1}{12}$ of 24, or 2. 100 % of the number is 100 × 2, or 200.

18. 28 is 7 % of what number? 4 %? 10 %? 14 %? 5 %?

19. 18 is 9 % of what number? 3 %? 6 %? 10 %? 2 %?

20. 24 is 6 % of what number? 12 %? 4 %? 8 %? 3 %?

21. 30 is 30 % of what number? 10 %? 15 %? 5 %?

22. 75 is 75 % of what number? 25 %? 5 %? 15 %?

23. 60 is 10 % of what number? 6 %? 5 %? 15 %?

24. 100 is 25 % of what number? 50 %? 10 %? $12\frac{1}{2}$ %?

25. 33 is 11 % of what number? 3 %? 10 %?

26. 40 is 10 % of what number? 5 %? 8 %? 20 %?

27. 21 is 3 % of what number? 7 %? 10 %? $12\frac{1}{2}$ %?

28. 45 is 5 % of what number? 9 %? 10 %? $33\frac{1}{3}$ %?

The above analysis requires the application of the first rule.

WRITTEN

Solve the following examples by the second rule:

29. 15 is 7 % of what number?

$$\begin{array}{r} 2\ 14\frac{2}{7} \\ \text{,07.})\overline{\text{15,00.}} \end{array}$$

30. 19 is 7 % of what number?

31. 256 is 16 % of what number? 8 %? 10 %?

32. 250 is 12 % of what number? 15 %? 18 %?

33. 12.5 is 4 % of what number? 1 %? 10 %?

34. 3.25 is 6 % of what number? 10 %? $12\frac{1}{2}$ %?

35. 46.8 is 8 % of what number? 16 %? 20 %?

Give the numbers of which the number at the left in each line is a given per cent:

36. \$18.50 is 20 %, $33\frac{1}{3}$ %, $66\frac{2}{3}$ %, of what number?

SOLUTION. \$18.50 is 20 %, or $\frac{1}{5}$, of \$92.50.

37. \$12.36 is $8\frac{1}{3}$ %, 75 %, 40 %, 60 %, 80 %, $37\frac{1}{2}$ %, of what number?

38. \$24.30 is $66\frac{2}{3}$ %, $33\frac{1}{3}$ %, $62\frac{1}{2}$ %, $16\frac{2}{3}$ %, $12\frac{1}{2}$ %, of what number?

39. $\frac{3}{4}$ is 20 %, 25 %, 30 %, $37\frac{1}{2}$ %, 10 %, $12\frac{1}{2}$ %, of what number?

Finding a number when a number increased or decreased by a certain per cent of itself is given.

STUDY RECITATION

If a number is *increased* by $\frac{1}{4}$ of itself, it equals $1\frac{1}{4}$ times the original number, or $\frac{5}{4}$ of it.

If a number is *decreased* by $\frac{1}{4}$ of itself, it equals $\frac{3}{4}$ of the original number.

1. *a.* If $\frac{5}{4}$ of a number is 25, what is the number?

b. If 125 % of a number is 25, what is the number?

c. If a number increased by 25 % of itself equals 25, what is the number?

2. *a.* If $\frac{3}{4}$ of a number is 18, what is the number?

b. If 75 % of a number is 18, what is the number?

c. If a number decreased by 25 % of itself equals 18, what is the number?

3. What number increased by 20 % of itself equals 18? 24? 30? 60? 90?

4. What number decreased by $33\frac{1}{3}$ % of itself equals 4? 8? 12? 16? 20? 24?

5. What number increased by $12\frac{1}{2}$ % of itself equals 18? 27? 45? 36? 54? 63? 72? 81?

6. What number increased by 6 % of itself equals 318?

100 % + 6 % = 106 %

$$1.06\overline{)318.00}$$

Since 318 equals 1.06 times the number, the number equals 318 ÷ 1.06.

Divide the number expressing the part by 100 % *plus the given rate per cent.*

7. What number decreased by 5 % of itself equals 380?

100 % − 5 % = 95 %

4 00

95.)380,00.

Since 380 equals .95 of the number, the number equals 380 ÷ .95, or 400.

Divide the number expressing the part by 100 % *minus the given rate per cent.*

WRITTEN

What number *increased* by	What number *decreased* by
8. 18 % of itself = 472?	28. 18 % of itself = 410?
9. 12½ % of itself = 810?	29. 12½ % of itself = 700?
10. 35 % of itself = $1080?	30. 35 % of itself = $2600?
11. 16 % of itself = $3480?	31. 16 % of itself = $7560?
12. 15 % of itself = $2300?	32. 15 % of itself = $6800?
13. 20 % of itself = $1800?	33. 25 % of itself = $6000?
14. 25 % of itself = $2500?	34. 20 % of itself = $1800?
15. 37½ % of itself = $2200?	35. 37½ % of itself = $5500?
16. 40 % of itself = $1400?	36. 5 % of itself = $3800?
17. 62½ % of itself = $2600?	37. 7 % of itself = $1860?
18. 87½ % of itself = $3000?	38. 8 % of itself = $1840?
19. 5 % of itself = $2100?	39. 9 % of itself = $9100?
20. 6 % of itself = $3180?	40. 10 % of itself = $1800?
21. 10% of itself = $3300?	41. 11 % of itself = $8900?
22. 15 % of itself = $3450	42. 28 % of itself = $7200?
23. 18 % of itself = $2360?	43. 50 % of itself = $1500?
24. 32 % of itself = $2640?	44. 60 % of itself = $4000?
25. 14 % of itself = 2280?	45. 35 % of itself = 6500?
26. 16 % of itself = 3480?	46. 40 % of itself = 6000?
27. 19 % of itself = 2380?	47. 18 % of itself = 8200?

PROFIT AND LOSS

The application of percentage to (1) profit and loss, (2) trade discount, (3) commission, (4) insurance, (5) taxes, and (6) duties involves nothing new except terms and business usages.

The **cost** of anything is the amount paid for it by the buyer.

The **selling price** is the amount received by the seller.

The **gain** or **profit** is the excess of the selling price over the cost.

The **loss** is the excess of the cost over the selling price.

The **list price** of an article is the amount asked for it.

The gain or loss per cent is always reckoned on the cost price. The cost is 100 % of itself.

Study Recitation

1. A merchant bought a coat for $15 and sold it for $20. How much did he gain? What per cent did he gain?

2. A merchant bought a coat for $15 and sold it for $10. How much did he lose? What per cent did he lose?

3. A merchant gained $5 by selling a coat for $20. What was the cost?

4. A merchant lost $5 by selling a coat for $20. What was the cost?

5. I bought a horse for $120 and sold it at a profit of 25 %. Find the gain and the selling price.

Analysis by Direct Statement

25 % of the cost = ¼ of it.

¼ of $120 = $30, gain.

$120 + $30 = $150, selling price.

Analysis by Question and Answer

What is 25 % of $120? *Ans.* $30.

$30 = gain. If a horse was bought for $120 and sold at a profit of $30, what was the selling price? *Ans.* $150.

6. I bought a horse for $120 and sold it for $150. Find the gain per cent.

ANALYSIS BY DIRECT STATEMENT

$150 − $120 = $30, gain.
$30 is $\frac{30}{120}$, or $\frac{1}{4}$, of $120.
$\frac{1}{4}$ of 100% = 25%, gain.

ANALYSIS BY QUESTION AND ANSWER

If a horse is bought for $120 and sold for $150, what is the gain? *Ans.* $30. $30 is what per cent of $120? *Ans.* 25%.

7. I sold a horse for $150, thereby gaining 25%. Find the cost.

ANALYSIS BY DIRECT STATEMENT

100% of the cost + 25% of it = 125% of the cost = selling price, $150.
125% of the cost = $\frac{5}{4}$ of it.
$\frac{5}{4}$ of the cost = $150.
$\frac{1}{4}$ of the cost = $4 \times \frac{1}{5}$ of $150, or $120, cost.

ANALYSIS BY QUESTION AND ANSWER

If a horse is sold at a gain of 25%, for what per cent of the cost is it sold? *Ans.* 125%. 125% of a number is what fraction of it? *Ans.* $\frac{5}{4}$ of it. $150 is $\frac{5}{4}$ of what number? *Ans.* $120. $120 = cost.

TO THE TEACHER. Lead pupils to see what is given and what is asked for in each of the three problems above, and how each problem may be derived from either of the others. Require pupils to give original problems like any one of the three, and then to derive from these original problems two others like those above.

PROBLEMS

ORAL

Each of the following problems may be changed so as to become a problem under one or more of the three general problems in percentage, decimals, or fractions.

1. A lot costing $1600 is sold at a profit of 12½%. What is the gain? the selling price?

2. A 50-cent knife is sold for 70 cents. Find the gain per cent.

3. If 100% is gained by selling a lace collar for $1.50, how much would be gained by selling it for $1?

4. A jeweler sold a watch for $32, thereby gaining $33\frac{1}{3}\%$. Find the gain and the cost.

5. By selling corn at 54 cents a bushel, I lost 10%. Find the cost and the loss.

6. What per cent do I lose when I buy at 20 cents and sell at 10 cents? What per cent do I gain when I buy at 10 cents and sell at 20 cents?

7. A 60-acre farm costing $4800 is sold at a gain of $12\frac{1}{2}\%$. Find the selling price.

8. A merchant buys pianos at $250 apiece. How much must he charge for them in order to make a profit of 20%?

9. What must be the list price of a watch costing $36 in order that a profit of 25% may be made on it?

10. What is the per cent of profit on apples bought for $1.50 a bushel and sold for $2.40 a bushel?

Written

11. For how much must I sell apples costing $2.40 a barrel to gain $37\frac{1}{2}\%$?

12. I lose 30% in selling a suit for $21.70. Find the cost.

13. I lose 10% in selling an article for $15.30. What would have been the selling price had I gained 10%?

14. A house costing $3750 is sold for $3975. What is the gain per cent?

15. An article costing $7.50 is sold at a loss of 20%. The purchaser sells it for $10. What is his gain per cent?

16. A piano costing $150 is sold at a profit of 20%. The retail merchant who buys the piano pays $5 freight, and sells it at a profit of 25%. Find the selling price.

17. A merchant sells carpet at \$.75 a yard, and gains 25%. What would be the gain if the carpet were sold at \$.85 a yard?

18. By selling an article for \$7.20 I gain 12½%. What per cent should I gain by selling it for \$7.50?

19. A dealer sells two wagons for \$75 each. On one he gains 25%, and on the other he loses 25%. Does he gain or lose by the transaction, and how much?

ANALYSIS BY QUESTION AND ANSWER

A gain or a loss of 25% is what part of the cost? *Ans.* ¼ the cost. If a gain of ¼ the cost is made by selling a wagon for \$75, \$75 is what fraction of the cost? *Ans.* ⅘ of the cost. \$75 is ⅘ of what sum? *Ans.* \$60. \$60 = cost.

If a loss of ¼ the cost results from selling a wagon for \$75, \$75 is what fraction of the cost? *Ans.* ⅗ of the cost. \$75 is ⅗ of what sum? *Ans.* \$100, or cost. (\$100 + \$60), cost, − \$150, selling price, = \$10, loss.

20. I sell two machines at \$150 each. On the first I gain 25%, and on the second I lose 20%. Do I gain or lose, and how much?

21. A boy buys 6 apples for a dime, and sells them at the rate of 2 for 5¢. What is his gain per cent?

22. A boy buys apples at the rate of 5 for 5¢, and sells them at the rate of 3 for 5¢. What is his per cent of gain?

23. A boy buys 120 oranges at the rate of 4 for 10¢, and sells them at the rate of 3 for 10¢. What is his per cent of gain or loss? What is his per cent of gain or loss if 10% of the oranges are unsalable?

24. A farmer sells 75 sheep for what 100 cost. Find his gain per cent.

SUGGESTION. 75 sheep cost ¾ as much as 100 sheep; therefore he gains ¼ of the cost.

25. A merchant buys bankrupt stock at 45¢ on the dollar, and sells it at 10% below the original list price. What is his gain per cent? What is meant by "buying at 45¢ on the dollar"?

26. An article costing $10 is sold for $8.50. What per cent is lost? Another article costing $8.50 is sold for $10. What per cent is gained?

27. Mr. Lewis bought 12,000 bushels of wheat at $.60 a bushel, and 6500 bushels of corn at $.55 a bushel. In selling, he gained $16\frac{2}{3}\%$ on the wheat and lost 20% on the corn. What per cent did he gain or lose on the whole transaction?

28. At what price must goods costing $40 be listed so that if they are sold at $16\frac{2}{3}\%$ below the list price a profit of 25% may be realized?

SUGGESTION. At what price must goods costing $40 be sold to realize a profit of 25%? *Ans.* $50. If $50 is received for goods sold at $16\frac{2}{3}\%$ below the list price, what is the list price? $50 is $\frac{5}{6}$ of what number?

29. A quantity of wheat which cost $1.16 per bushel was sold at a loss of 25%. How many bushels were sold if the total loss was $37.70?

SUGGESTION. (*a*) What is the loss on 1 bushel? Then $37.70 is the loss on how many bushels?

(*b*) If $37.70 is $\frac{1}{4}$ of the total cost, what is the total cost? Then how many bushels at $1.16 can be bought for this amount?

30. Fifty pounds of soap are purchased at 5 cents a pound. In a damp cellar it absorbs water equal to 10% of its weight, and is then sold at 6 cents a pound. What is the gain? the gain per cent?

31. I sell $\frac{3}{5}$ of an article for what $\frac{3}{4}$ of it costs. Find the gain per cent.

SUGGESTION. I sell $\frac{1}{5}$ of the article for $\frac{1}{4}$ of the cost and $\frac{2}{5}$ of it for $\frac{2}{4}$ of the cost.

COMMERCIAL OR TRADE DISCOUNT

STUDY RECITATION

Manufacturers and merchants frequently deduct a certain per cent from the list price of goods when selling them or offering them for sale. This deduction is called a **discount.**

Find the selling price of the following goods offered at a bargain sale:

		REGULAR PRICE	DISCOUNT SALE PRICE
1.	Cloth	\$1.60 a yd.	$12\frac{1}{2}\%$
2.	Gowns	\$35	20%
3.	Handkerchiefs	\$2.00 a doz.	25%
4.	Suits	\$48	$37\frac{1}{2}\%$
5.	Umbrellas	\$4.00	30%
6.	Shoes	\$5.00	10%
7.	Stockings	\$1.25	20%
8.	Clocks	\$6.00	$66\frac{2}{3}\%$
9.	Blankets	\$8.00	$37\frac{1}{2}\%$

Many business houses give two or more discounts off the list price in consideration of payment in cash, changes in market price, or for other reasons.

These deductions are called **trade discounts** or **commercial discounts.**

NOTE. Discounts made for payments in cash are sometimes called **cash discounts**, and discounts made for payment within a certain time, as 10 days, 1 month, etc., are called **time discounts.**

The **net price** is the list price less the discounts; it is the price the buyer pays.

In trade discounts, the first discount is computed on and deducted from the list price; the second discount is computed on and deducted from the resulting remainder, etc.

10. A piano was listed at \$600 with a discount of $33\frac{1}{3}\%$ and another discount of 10 % for cash. Find the net price.

3)\$600	$33\frac{1}{3}$ of a number = $\frac{1}{3}$ of it.
200	$\frac{1}{3}$ of \$600 = \$200, first discount.
10)400	\$600 − \$200 = \$400, remainder.
40	10 %, or $\frac{1}{10}$, of \$400 = \$40, second discount.
\$360	\$400 − \$40 = \$360, net price. *Ans.*

Or, a discount of $33\frac{1}{3}\%$, or $\frac{1}{3}$, of the cost, leaves $\frac{2}{3}$ of the cost. A further discount of 10 %, or $\frac{1}{10}$, of the remainder, leaves $\frac{9}{10}$ of $\frac{2}{3}$, or $\frac{3}{5}$, of the cost. Net price = $\frac{3}{5}$ of \$600 = \$360.

Written

Find the net price of each of the following sales:

	List Price	Discounts		List Price	Discounts
11.	\$60	25 %, 20 %	**15.**	\$600	$16\frac{2}{3}\%$, 5 %
12.	\$125	20 %, 25 %	**16.**	\$800	$12\frac{1}{2}\%$, 6 %
13.	\$180	$16\frac{2}{3}\%$, 20 %	**17.**	\$840	20 %, $37\frac{1}{2}\%$
14.	\$270	$33\frac{1}{3}\%$, 25 %	**18.**	\$1000	50 %, 10 %

19. The following bill was sold at a discount of 20 %:

Indianapolis, Ind., Sept. 2, 1914.

C. B. Baxter,

Menomonie, Wis.,

Bought of ACME TOOL COMPANY

Terms: 30 days net; 5% 10 days.

2 doz. hammers	@ \$ 5.00	\$10.00	
2 doz. handsaws	@ \$12.00	24.00	
		34.00	
	Less 20 %	6.80	
			\$27.20
	Less 5 %		1.36
Rec'd payment, Acme Tool Company per J.			\$15.84

Mr. Baxter secured the discount of 5 % by paying his bill within 10 days. If he had paid in 30 days, his bill would have been \$27.20.

PROBLEMS

WRITTEN

1. Discounts of 20 % and 10 % are allowed on goods listed at $60. Find the discounts and the net price.

2. A gun was listed at $120, with trade discounts of $33\frac{1}{3}$ %, 25 %, and 10 %. What was the selling price?

3. A merchant sold a bill of goods listed at $200 with 20 % and 5 % off for cash. How much did he receive?

4. Which is better for the purchaser, 40 % and 20 % off, or a single discount of 53 %?

5. Find a single discount equivalent to 25 %, 20 %, and 10 % off, on a bill of goods listed at $120.

6. A merchant buys goods at 50 % and 20 % off, and sells them at 10 % and 20 % off the list price. What is his gain per cent?

7. A merchant bought 40 yd. of silk listed at 83 ¢ a yard, at a discount of 50 % and 10 % off. He sold the silk at 25 % and 10 % off from the list price. What was his profit?

8. What is the net amount of a bill, the list price of which is $500, trade discount 10 %, and 5 % off for cash?

9. A bill of goods amounted to $3300, but the buyer was allowed $16\frac{2}{3}$ % and 10 % off. How much did he pay?

10. G. H. King & Co., of Evansville, Ind., sells to L. N. Reed of Chicago, Ill., on account, 60 days net, 5 % 10 days: 5 sewing machines at $60 each, less 20 % discount; and 1 doz. sewing machines at $50 each, less 25 % discount. Make out, foot, and receipt the bill, and find the net amount if it is paid within 10 days; if it is not paid for 60 days.

11. Construct and solve two other problems involving trade discount.

COMMISSION AND BROKERAGE

An **agent** is a person authorized to buy, sell, or transact business for another.

An agent who buys or sells goods is often called a **commission merchant.**

An agent who buys and sells real estate, bonds, and stocks for others is called a **broker.**

Commission is a per cent paid to an agent for the transaction of business. The commission paid to a broker is often called **brokerage.**

The **proceeds** is the sum left after the commission and other expenses have been deducted.

Business usage or principle.

In selling, commission is computed on the sum received for the goods; in buying, on the sum paid for the goods; in collecting, on the sum collected.

For information concerning the commission, or brokerage, paid a broker for buying or selling stocks and bonds, see p. 376.

STUDY RECITATION

1. An agent sells goods to the amount of $2450. What is his commission at $2\frac{1}{2}\%$, and what are the proceeds?

SOLUTION

$2\frac{1}{2}\%$ of a number $= .02\frac{1}{2}$ of it.
$\$2450 \times .02\frac{1}{2} = \$61.25$, commission.
$\$2450 - \$61.25 = \$2388.75$, proceeds.

2. If an agent receives $500 for selling goods at 5 % commission, how much does he sell?

SOLUTION

$500 is 5% of what number? $\$500 \div .05 = \$10{,}000$. *Ans.*

PROBLEMS

Oral

1. An agent sold a house for $6000 at 10% commission. What was his commission?

2. Find the commission at 2% on 1500 bu. of wheat sold at $1 a bushel.

3. If I receive $1000 for buying goods at 10% commission, what is the cost of the goods?

4. How much better is a commission of 6% on $1000 than a commission of 2% on $1500?

Written

5. An agent collected a $565 debt. What was his commission at 5%, and how much should he remit to the creditor?

6. Mr. B. sold 16,000 bu. of wheat at $.75 a bushel on a commission of $2\frac{1}{2}$%. What was his commission?

7. A merchant received $61.25 as his commission for selling goods to the amount of $2450. What was the rate of his commission?

8. A real estate dealer bought 6 lots at $2200 each, and charged 5% commission. What was his commission?

9. An agent charged 5% commission for selling a quantity of goods. He received $15 for his services. What was the selling price of the goods? How much did the party receive for whom the goods were sold?

Analyze the problem beginning with: 5% of the selling price = $15.

10. A commission merchant sold 25 boxes of oranges each containing 12 dozen, at $.25 per dozen. Find his commission at 6%.

11. Mr. Cook planted 10 acres in celery, paying $60 per acre for labor, etc., and $50 per acre for fertilizers. He cut from each acre 27,000 plants, which he sold in crates, each crate containing 5 dozen bunches of 3 plants each at $3.00 per crate, with a commission of 4% for selling. Find Mr. Cook's profit on the 10 acres of celery.

12. A commission merchant receives turkeys weighing 3140 lb. He sells them at 11¢ a pound. How much is his commission at 5%?

13. If a commission merchant receives $2\frac{1}{2}$% commission for buying goods, how much does each dollar's worth of goods cost his principal, or the party for whom he is acting?

14. A real estate agent is offered a yearly salary of $1800, or a commission of $2\frac{1}{2}$% on his sales. If his average monthly sales amount to $4800, which is the better offer for him to accept? What is the difference between the two offers for one year?

15. A commission firm is ordered to buy 5000 bushels of wheat @ 75¢. The commission is $\frac{1}{8}$¢ per bushel. What is the cost of the wheat, including the commission?

16. Find the cost of 500 baskets of peaches at 70¢ a basket, the commission for buying being $4\frac{1}{2}$%.

17. Find the total cost of 1000 bales of cotton weighing 500 lb. each, at $.08$\frac{3}{4}$ a pound, the freight being $.50 a hundred pounds, drayage $70, and commission 3%.

18. An agent for an automobile company received a commission of 10% on his sales. In one month he sold one automobile at $900 and another at $1250. How much did he earn? How much did he make above his expenses of $75?

19. Construct and solve two problems in commission.

INSURANCE

Insurance is security against financial loss on account of destruction or loss of property, health, or life. We shall consider property insurance only.

The **policy** is the contract between the insured and the insurance company.

The **face** of the policy is the amount insured.

The **premium** is the sum paid the insurance company for taking the risk.

The **rate** is a specified sum per $100 of insurance or a certain per cent of the sum insured.

Business usage or principle.

Premiums are computed on the face of the policy, usually at a certain sum per hundred dollars. Thus, a premium of 80¢ per hundred for three years means that the company charges 80¢ for every $100 insurance for three years.

Study Recitation

1. A house valued at $24,000 is insured for three years for $\frac{2}{3}$ of its value at 80¢ a hundred. What is the amount of premium to be paid?

Solution.

$\frac{2}{3}$ of $24,000 = $16,000, face of policy.

$\frac{\$16,000}{100} \times .80 = \$128$, premium.

PROBLEMS

Oral

1. How much must I pay for a policy of $1000 on my furniture at 50¢ a hundred for one year?

2. A ship worth $80,000 was insured for $\frac{7}{8}$ of its value at $1.00 per hundred. Find the premium.

WRITTEN

3. Mr. Brown insured his house for $3500, his barn for $850, and his personal property for $1250, at 90 ¢ a hundred for three years. Find the premium.

4. Find the cost of insuring a piano worth $440 for full value at $\frac{3}{4}\%$.

5. Find the cost of insuring a house worth $12,000 for $\frac{2}{3}$ of its value at $\frac{1}{2}\%$.

6. I insured a house worth $24,800 for $\frac{7}{8}$ of its value at $\frac{7}{8}\%$. Find the premium.

7. A building valued at $40,000 is insured for three years for $\frac{7}{8}$ of its value, at 80 ¢ a hundred. What is the amount of the premium to be paid?

8. A factory was insured for $15,000, the rate of premium being $40 per $1000. After 3 annual premiums had been paid, a fire occurred. The stock saved was valued at $4800. How much insurance did the owner realize after deducting the cost of insurance?

9. A company charges a premium of $39.20 on $4900. What is the premium per hundred? the rate per cent?

10. An insurance company whose rate is 80 ¢ a hundred charges Mr. Goodyear $278.20 premium. Find the face of the policy.

11. After three annual payments of $500 had been made on a factory, the premium being at 2% on $\frac{7}{8}$ of the value of the factory, it was burned. Find the loss to the insurance company. What was the owner's loss by the fire?

12. Construct and solve two problems in insurance.

TAXES

A **tax** is a sum of money assessed on property for public use.

The **assessed valuation** of property is its estimated value for the apportionment of taxes.

The **rate of taxation** is the amount of tax per dollar of assessed valuation. Thus, a rate of .0285 means that $.0285 are charged on every dollar of assessed valuation.

Principle.

The tax is computed on the assessed valuation.

Study Recitation

1. In a township whose assessed valuation is $625,000, a tax of $15,000 is to be raised. What is the rate of taxation, and what is A's tax if his property is assessed at $10,500?

Solution

$15,000 is $\frac{15000}{625000}$, or $\frac{3}{125}$, of $625,000.

$\frac{3}{125}$ of 100% = $2\frac{2}{5}$%, rate of taxation.

$2\frac{2}{5}$% of a number = .024 of it, expressed as the decimal of a dollar. $.024 is the rate of taxation in cents and mills per dollar.

$10,500 × .024 = $252, A's tax.

PROBLEMS

Oral

1. What is B's tax if the rate of taxation is $1\frac{1}{2}$% and his assessed valuation is $1000?

2. In a certain city whose assessed valuation is $1,000,000, a tax of $10,000 is to be raised. Find the rate of taxation and Mr. Healy's tax on property assessed at $6000.

3. In a certain township the tax was $.017 on a dollar. What was the tax on a farm assessed at $10,000?

WRITTEN

4. Which would be more profitable for me, and how much more profitable, — to rent my house for $50 a month, taxes and repairs costing $140 annually; or to sell it for $5000 and invest the money at 5 %?

5. In a township whose assessed valuation is $900,000, a tax of $10,000 is to be raised. What is the rate of taxation, expressed in cents and mills?

6. Mr. Field owns property worth $8500. He pays yearly $100 taxes, $50 for repairs, and $30 for other expenses. For how much per month must he rent his property in order to make 4 % on his investment?

7. The tax rate in a city is 1.4 %. Find the yearly tax on a house and lot valued at $6700 and furniture worth $300.

8. A town has an assessed valuation of $4,500,000 and wishes to levy a tax of $35,000 for bridges. What must be the rate of taxation?

9. Construct and solve two problems in taxes.

FEDERAL INCOME TAX

The income tax law of the United States, passed in 1913, requires that every single person shall pay an annual tax of 1 % on his net income in excess of $3000, and married persons living together an annual tax of 1 % on their joint income in excess of $4000. When the net income exceeds $20,000, an additional tax, or **surtax**, thereon must be calculated as per schedule below:

1 % on amount over $20,000 and not exceeding $50,000.
2 % on amount over $50,000 and not exceeding $75,000.
3 % on amount over $75,000 and not exceeding $100,000.
4 % on amount over $100,000 and not exceeding $250,000.
5 % on amount over $250,000 and not exceeding $500,000.
6 % on amount over $500,000.

Study Recitation

1. Albert Blake and his wife together have a gross income as follows: Salary, $5000; interest on bonds, $1000; rent on house, $1500. To find their net income, Mr. Blake deducts the following expenses: Taxes, $200; repairs, $100; interest on mortgage, $500. On his net income he is allowed an exemption of $4000. What is his income tax on the remainder at 1%?

Solution. Gross income	$7500
General deductions	800
Net income	6700
Exemption	4000
Taxable income on which 1% is to be calculated	$2700

Income tax, 1% of $2700 = $27.

2. Find the income tax of a man living with his wife who has a net income of $110,000.

Solution. $110,000 − $4000 exemption = $106,000		
Normal tax of 1% on $106,000	=	$1060
Surtax of 1% on ($50,000 − $20,000) $30,000	=	300
Surtax of 2% on ($75,000 − $50,000) $25,000	=	500
Surtax of 3% on ($100,000 − $75,000) $25,000	=	750
Surtax of 4% on ($106,000 − $100,000) $6000	=	240
		$2850

Written

Find the Federal income tax of a man whose exemption is $4000 on a net income of:

3. $4500 5. $25,000 7. $60,000 9. $120,000 11. $240,000
4. $6500 6. $80,000 8. $76,000 10. $180,000 12. $510,000

Find the Federal income tax of a woman whose exemption is $3000 on a net income of:

13. $4600 15. $15,000 17. $79,000 19. $190,000 21. $250,000
14. $3500 16. $28,000 18. $58,000 20. $260,000 22. $520,000

DUTIES

Duties or **customs** are taxes levied by the government on imported goods.

An **ad valorem duty** is a tax computed upon the price paid for the goods; as, an ad valorem duty of 75% on the cost.

A **specific duty** is a tax on the quantity of goods without regard to their cost; as, a specific duty of 15 cents a yard, or 25 cents a pound.

An **invoice** is a bill accompanying the imported goods, showing their quantity and value.

A **tariff** is a schedule of duties.

NOTE. Duties are collected at **customhouses** which are found at ports of entry.

Principle.

Ad valorem duties are computed at a certain per cent of the value of the goods, and specific duties at a certain sum per unit of weight, length, etc.

STUDY RECITATION

1. What is the duty, at 30% ad valorem, on 16,500 clocks invoiced at $1.60 each?

SOLUTION

16,500 × $1.60 = $26,400, cost.
30% of a number = .30 of it.
$26,400 × .30 = $7920, duty.

2. What is the duty, at $1.85 a pound, on 45 cases of tobacco of 185 lb. each?

SOLUTION

45 × 185 lb. = 8325 lb.
8325 × $1.85 = $15,401.25

PROBLEMS

WRITTEN

1. Find the duty on 5000 bu. of oats and 10,000 bu. of barley, the duty on oats being $.06 per bushel and on barley $.15 per bushel.

2. S. M. Giles of Chicago imported from Geneva 150 watches invoiced at $55 each, and 250 diamonds at $80 each. What was the duty, the rate being 30% ad valorem on watches and 10% on diamonds?

3. Find the duty, at 20% ad valorem, on 2260 lb. of cheese invoiced at 40¢ a pound.

4. What is the duty, at 25% ad valorem, on blankets valued at $225?

5. Find the duty, at 50% ad valorem, on a rug valued at $2800.

6. Stewart & Co. imported from England 18 doz. silk umbrellas invoiced at $2 each. How much must they pay for the invoice, including ad valorem duty at 35%?

7. Find the duty, at 25% ad valorem, on an invoice of $300 worth of blankets.

8. What is the duty, at 30% ad valorem, on 500 dozen table knives valued at $7 a dozen?

9. J. Ransom & Co. imported from Havana 75 hhd. of molasses, at $5 a hogshead; and 800 boxes of cigars (each box weighing 1 lb. and containing 100 cigars), at $8 per box. 7% was allowed for leakage on the molasses. Find the duty at 15% ad valorem on the molasses, and $4.50 per pound plus 25% ad valorem on the cigars.

10. Construct and solve two problems in customs and duties.

REVIEW

1. Define per cent; cost; selling price; profit; loss; trade discount; cash discount; net price; commission; agent; proceeds; insurance; policy; premium; tax; assessed valuation; specific duty; ad valorem duty; invoice; tariff.

2. Give the rules for expressing per cents as common fractions and common fractions as per cents.

Problems without Numbers

3. How can you find any per cent of a number?

4. How can you find what per cent one number is of another?

5. How can you find a number when a certain per cent of it is given?

6. How can you find a number when the number increased or decreased by a certain per cent of itself is given?

7. If you know the cost and the selling price how can you find the gain or the loss per cent?

8. If you know the selling price and the per cent of gain or loss how can you find the cost?

9. If you know the gain and the per cent of gain how can you find the cost? the selling price?

10. If you know the list price and several rates of discount, how can you find the net price?

11. If you know the amount of a sale and the rate of commission, how can you find the amount of commission?

12. If you know the amount insured and the rate of insurance, how can you find the premium?

13. If you know the assessed valuation and the rate of taxation, how can you find the amount of tax?

14. If you know the amount of an invoice and the rate ad valorem, how can you find the amount of duty?

GENERAL REVIEW

WRITTEN

Time yourself with these exercises. Then work them again, trying to beat your record.

Write from dictation, add, and test results.

	a.	*b.*	*c.*	*d.*
1.	978609	825499	$5438.12	$5382.49
	183214	943888	9321.42	3892.38
	536189	797677	4432.65	4781.44
	741062	684369	7567.83	1672.55
	387418	432408	6098.74	2563.66
2.	$38\frac{5}{8}$	$23\frac{3}{10}$	$87\frac{5}{8}$	$42\frac{5}{6}$
	$32\frac{3}{4}$	$44\frac{1}{16}$	$60\frac{5}{16}$	$25\frac{2}{3}$
3.	1.2475	32.08	384.9877	990.09
	43.8054	1.043	84.456	43.
	56.0205	942.987	469.234	674.0004
	75.7189	853.897	794.0089	.0005
4.	3 ft. 9 in.	5 pk. 7 qt.	5 lb. 4 oz.	4 doz. 3
	5 ft. 6 in.	6 pk. 3 qt.	3 lb. 3 oz.	5 doz. 9

Subtract and test:

5.	874321	453621	$9008.95	$1945.72
	595698	321069	5123.48	1134.54
6.	$56\frac{1}{4}$	$39\frac{2}{3}$	$74\frac{5}{8}$	31
	$40\frac{1}{8}$	$18\frac{5}{6}$	$62\frac{3}{16}$	$15\frac{1}{6}$
7.	987.04	87.0802	518.	418.056
	34.5109	49.34	.0762	219.8

	a.	*b.*	*c.*
8.	16 yd. 2 ft.	20 mi. 8 rd.	15 wk. 2 da.
	12 yd. 2 ft.	19 mi. 16 rd.	8 wk. 5 da.

Find the products, canceling when possible:

	a.	*b.*	*c.*	*d.*
9.	953×8742	$\frac{3}{4} \times \frac{5}{8}$	5.06×3.087	$215 \times \$64.18$
10.	278×6209	$\frac{5}{16} \times \frac{5}{6}$	$.034 \times 1.0275$	$23\frac{1}{3} \times 14\frac{2}{5}$
11.	$1650 \times \$328.56$	$25\frac{1}{4} \times 13\frac{1}{3}$	$.07 \times .0057$	1.009×25.08

Find the quotients and remainders. Test answers.

	a.	*b.*	*c.*	*d.*
12.	$80565 \div 615$	$\frac{5}{8} \div \frac{3}{4}$	$.875 \div .025$	4 ft. 6 in. $\div 4$
13.	$908800 \div 568$	$\frac{3}{5} \div \frac{3}{4}$	$38.844 \div 16.6$	4 gal. 1 qt. $\div 8$
14.	$\$6318.90 \div 900$	$15\frac{5}{6} \div 3\frac{1}{6}$	$388.44 \div 2.34$	$.50463 \div .089$

15. Find the ratio of 12 to 48; of 48 to 12; of $\frac{3}{4}$ to $\frac{3}{8}$.

16. Find the ratio of 1 in. to 5 ft.; 5 ft. to 1 in.

Reduce to improper fractions, whole, or mixed numbers.

	a.	*b.*	*c.*	*d.*	*e.*
17.	$2\frac{1}{4}$	$3\frac{5}{8}$	$7\frac{1}{2}$	$5\frac{3}{5}$	$6\frac{3}{4}$
18.	$\frac{25}{3}$	$\frac{14}{7}$	$\frac{16}{8}$	$\frac{24}{4}$	$\frac{19}{5}$

19. Reduce to lowest terms: $\frac{15}{45}$; $\frac{20}{30}$; $\frac{16}{36}$; $\frac{15}{90}$; $\frac{27}{36}$; $\frac{32}{40}$.

20. Reduce to fractions having the l.c.d.: $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$; $\frac{3}{4}, \frac{3}{8}, \frac{3}{16}$; $\frac{2}{5}, \frac{3}{10}, \frac{1}{15}$; $\frac{3}{5}, \frac{1}{8}, \frac{1}{4}$.

Reduce common fractions to decimals and to per cents; and decimals and per cents to common fractions.

	a.	*b.*	*c.*	*d.*	*e.*
21.	$\frac{3}{4}$	$\frac{3}{16}$	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{4}{25}$
22.	.30	$.37\frac{1}{2}$	$.16\frac{2}{3}\%$	2.25	3.75%
23.	.75	$.87\frac{1}{2}$	$.83\frac{1}{3}\%$	1.50	$3.37\frac{1}{2}$

Reduce the numbers in (1) to the denominations in (2):

	a.		*b.*		*c.*	
	(1) Number	(2) Denomination	(1) Number	(2) Denomination	(1) Number	(2) Denomination
24.	8 qt.	pints	48 oz.	pounds	36 sq. ft.	sq. yards
25.	5 gal.	quarts	8000 lb.	tons	4 A.	sq. rods
26.	8 qt.	gallons	5 T.	pounds	54 cu. ft.	cu. yards
27.	16 qt.	pecks	24 in.	feet	4 cu. yd.	cu. feet
28.	8 pt.	quarts	3 ft.	inches	180 sec.	minutes

Find the cost of:

29.	164 lb. @ 25¢	18 yd. @ $83\frac{1}{3}$¢	10 yd. @ $1.50
30.	32 lb. @ $12\frac{1}{2}$¢	24 yd. @ $37\frac{1}{2}$¢	10 lb. @ $1.25
31.	18 lb. @ $33\frac{1}{3}$¢	48 yd. @ $16\frac{2}{3}$¢	8 gal. @ 1.62\frac{1}{2}$

Find the number of yards that can be bought:

	Amount	Price	Amount	Price	Amount	Price
32.	$2	$.10	$20	$.12$\frac{1}{2}$	$50	$.33$\frac{1}{3}$
33.	$5	$.50	$21	$.37$\frac{1}{2}$	$26	$.66$\frac{2}{3}$

	a. 10% of:	*b.* 33⅓% of:	*c.* 25% of:	*d.* 20% of:	*e.* 133⅓% of:	*f.* 187½% of:
34.	50	$240	200 T.	7340	24000	72000
35.	75	$150	500 cu. in.	6890	36000	96000
36.	100	$300	700 gal.	8250	87000	80000

Find what per cent the first number is of the second:

	a.	*b.*	*c.*	*d.*
37.	10 of 40	12 of 45	300 of 500	2 in. of 1 ft.
38.	40 of 10	45 of 12	500 of 300	1 oz. of 2 lb.

Of what number is

39.	18, 2%	10, 5%	$210, $37\frac{1}{2}$%	$2400, $37\frac{1}{2}$%
40.	24, 3%	16, 4%	$100, $62\frac{1}{2}$%	$1000, $16\frac{2}{3}$%

Find the profit or loss:

	a.		*b.*		*c.*	
	Cost	Rate of Gain	Cost	Rate of Loss	Cost	Rate of Gain
41.	$100	25%	$80	12½%	$500	8%
42.	$60	33⅓%	$200	18%	$700	4%

Find the per cent of profit or loss:

	Cost	Selling Price	Cost	Selling Price	Cost	Selling Price
43.	$100	$125	$50	$75	$30	$25
44.	$100	$75	$30	$35	$50	$40

Find the discount and the net price:

	List Price	Rate of Discount	List Price	Rate of Discount	List Price	Rates of Discount
45.	$100	25%	$300	16⅔%	$300	33⅓%, 10%
46.	$40	12½%	$600	25%	$2000	25%, 5%

Find the commission:

	Sales	Rate of Com.	Sales	Rate of Com.	Purchase	Rate of Com.
47.	$5000	20%	$1000	15%	$900	1½%
48.	$7200	12½%	$2000	18%	$8500	4%

Find the premium on each of the following policies:

	Face of Policy	Rate	Face of Policy	Rate	Face of Policy	Rate
49.	$1000	20¢ per $100	$8000	¼%	$20000	⅜%
50.	$4050	60¢ per $100	$1400	⅜%	$30000	1½%

Find the amount of taxes:

	Assessment	Tax Rate	Assessment	Tax Rate	Assessment	Tax Rate
51.	$2000	$.03	$1500	1½%	$8200	2¼ mills
52.	$3000	$.001	$2550	2½%	$3600	3½ mills

Find the duty on each of the following imports:

	Value	Ad val. Duty	Value	Ad val. Duty	Quantity	Specific Duty
53.	$1000	20%	$20000	25%	100 lb.	$1.50 a pound
54.	$5600	12½%	$16000	37½%	300 bu.	$.05 a bushel

PROBLEMS

Oral

1. Make change from \$5 for a purchase of $3\frac{1}{2}$ yd. ribbon @ 24¢ and $4\frac{1}{4}$ yd. lace @ 80¢.

2. How many acres are there in my farm if it is divided into three fields of 65 acres, 14 acres, and 29 acres?

3. There were 30 pupils in school on Monday. This number was $\frac{2}{3}$ of the total enrollment. How many pupils were enrolled?

4. What number increased by $.37\frac{1}{2}$ of itself equals 440?

5. What number decreased by $.37\frac{1}{2}$ of itself equals 500?

6. How many yards of lace at 8¢ a yard can be bought for 50¢?

7. William is $\frac{2}{7}$ as old as his sister. William's age is 14 years. How old is his sister?

8. A girl saves \$1.50 a week. Her savings are $\frac{3}{4}$ of her wages. How much does she receive per week?

9. A produce dealer sells corn at 60¢ a bushel, and gains $\frac{1}{5}$ of what it cost him. What is the cost per bushel?

10. If \$49 is $87\frac{1}{2}$ % of my money, how much money have I?

11. A farmer sold his farm for \$50 an acre, and gained 25 % of the cost. How much did it cost him per acre?

12. A second-hand book is sold for 20 % less than it cost. If it is sold for 40¢, what is the cost?

13. Oatmeal contains 15.6 % protein, and rice, 7.8 %. How many more pounds of protein are there in 100 lb. of oatmeal than in 100 lb. of rice?

14. Find the cost of 6 yd. of lining at $\$.12\frac{1}{2}$ a yard.

WRITTEN

15. A Camp Fire girl earned $10 per month. She spent $\frac{1}{4}$ of it for a hat, and $\frac{1}{10}$ of the remainder for gifts, and saved the rest. How much did she save in 5 months?

16. There are 1000 pupils enrolled in school. There are 669 present. How many are absent?

17. I buy 400 apples at the rate of 4 for 1¢ and sell them at the rate of 8 for 5¢. What is the gain?

18. Make out and receipt a bill for:

$5\frac{1}{2}$ yd. velvet @ $4.80
$2\frac{1}{4}$ yd. lace @ $.84
$3\frac{1}{4}$ doz. buttons @ $1.16
2 pairs gloves @ $1.50
4 yd. lining @ $.12$\frac{1}{2}$
$4\frac{5}{8}$ yd. silk @ $1.20

19. After spending .16$\frac{2}{3}$ of his money, Mr. Clark had enough left to buy 18 yd. of linen at 33$\frac{1}{3}$¢ a yard. How much linen could he have bought with all his money?

20. A stenographer's salary is $1000 a year. She spends $\frac{1}{4}$ of it for board and room, $\frac{1}{25}$ of it for laundry, $\frac{1}{4}$ of it for clothes, and $\frac{1}{10}$ of it for miscellaneous expenses. How much money does she save?

21. A class of 20 girls was provided with material for aprons. Each girl used $\frac{3}{4}$ yd. How much was needed by the class?

22. How much does Mrs. Green save by buying soap for $2.20 for 50 bars instead of buying it in separate bars at $.05 a bar?

23. Martha used for a cake 2 cups of sugar @ $2\frac{1}{2}\,¢$, $\frac{1}{4}$ lb. of butter @ $30\,¢$, 2 eggs @ $3\frac{1}{2}\,¢$ each, $\frac{1}{2}$ pt. milk @ $8\,¢$ a quart, 2 cups of flour worth $3\,¢$, and baking powder and salt worth $\frac{1}{2}\,¢$. How much did the cake cost?

24. Find the duty on $4500 worth of goods at $33\frac{1}{3}\%$ ad valorem.

25. If a suit was sold at $24.20, which was at a discount of 20% from the marked price, what was the marked price?

26. A man walked $\frac{5}{12}$ of his journey the first day, $\frac{3}{8}$ of it the next day, and then had 20 miles to travel. How long was the journey?

27. I spent $\frac{1}{4}$ of my money for groceries, 20% for house rent, $\frac{3}{8}$ for clothing, and the remainder, $133, for sundries. How much had I at first?

28. If a dress sold at $36.90 is sold at a profit of $12\frac{1}{2}\%$, what is the cost of the dress?

29. A piece of property worth $20,000 was taxed $2\frac{1}{2}\%$. What was the amount of the tax?

30. What per cent of 16 is 21? of 21 is 16? What per cent of $\frac{3}{8}$ is $\frac{7}{8}$? of $\frac{7}{8}$ is $\frac{3}{8}$?

31. What per cent of a yard is 2 ft. 9 in.?

32. What per cent of 8 is .375? of 1 ton is 75 lb.? of $27.50 is $12\frac{1}{2}$ cents?

33. A lost $33\frac{1}{3}\%$ of his money and then found that $16\frac{2}{3}\%$ of the remainder was $30 less than he lost. How much did he lose, and how much had he at first?

34. A merchant's debts are $10,500, and his property amounts to $8400. What per cent of his debts can he pay? How much will Mr. Grey receive, if he has a claim of $750?

35. What is the net price of a bill of goods for $8000 sold at discounts of 25% and 10%? at one discount of 30%?

36. What is the difference between $\frac{1}{8}\%$ of $720 and $33\frac{1}{3}\%$ of $720? Write both per cents as mixed decimals of two places.

37. Find the cost of insuring a house worth $64,000 for $\frac{7}{8}$ of its value at $\frac{1}{2}\%$.

38. Find the commission on a sale of $4600 worth of goods at $2\frac{3}{4}\%$.

39. If A's money is $33\frac{1}{3}\%$ more than B's, what per cent of A's money is B's?

40. If a man who fails in business owes $20,000 and has only $16,000 worth of property, what per cent of his indebtedness can he pay? If he settles with his creditors at $.20 on the dollar, how much of his assets will he save?

41. Mr. Burr delivered 1000 pounds of 3.5 per cent milk at a factory. Find the value of the butter fat at $.24 a pound.

42. If 92.4 lb. of butter fat were obtained from 2200 lb. of milk, what per cent did the milk test?

43. If milk that analyzed 4.5% butter fat yielded 900 lb. of butter fat, how many pounds of milk were there?

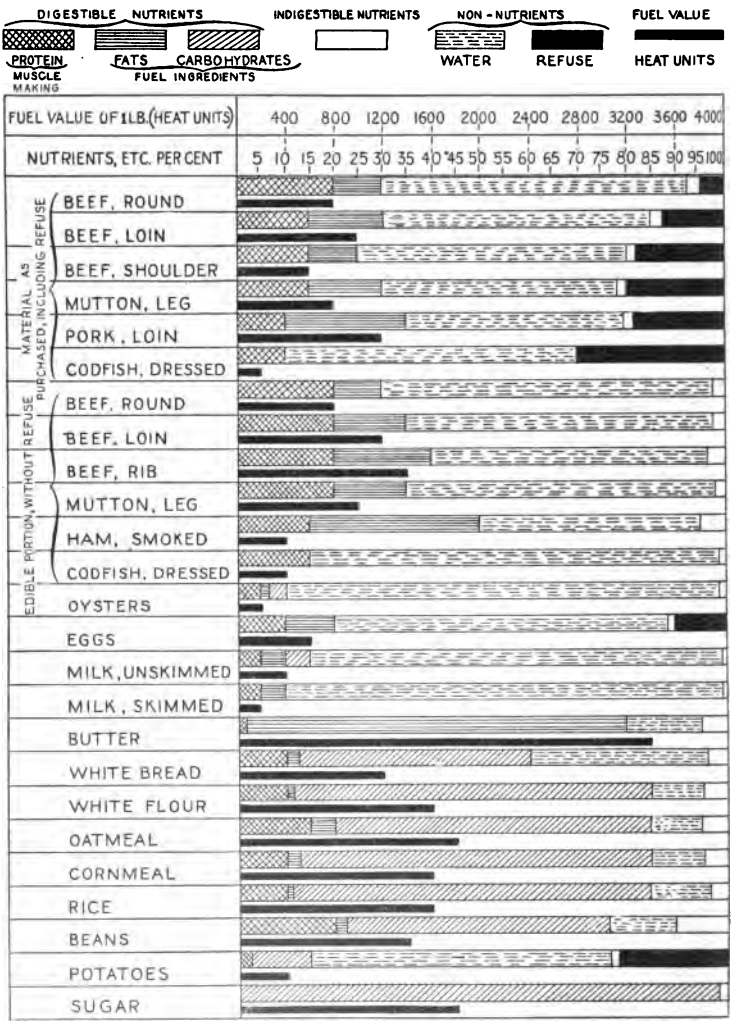
44. A boy who had 240 chickens sold 60 of them. What part remained unsold? What per cent?

45. A man paid $910 for 13% of a house. What was the value of the house at that rate?

46. A man spends in one year $1260, which is $66\frac{2}{3}\%$ of his income. What is his income?

COMPOSITION OF FOOD MATERIALS

Approximate Per cents of nutritive ingredients, refuse, and fuel value



Find, from the diagram on the opposite page, the number of pounds each of protein and fats in 10 pounds of:

47. Round beef as purchased, including refuse; of edible portion without refuse.

48. Loin of beef as purchased, including refuse; of edible portion without refuse.

49. Shoulder of beef as purchased, including refuse; of edible portion of rib of beef without refuse.

50. Leg of mutton as purchased, including refuse; of edible portion without refuse.

51. Loin of pork as purchased, including refuse; of edible portion of smoked ham without refuse.

52. Dressed codfish as purchased, including refuse; of edible portion without refuse.

Find the number of pounds of protein in 10 lb. of:

53. Oysters. **54.** Eggs. **55.** Rice. **56.** Beans.

Find the number of pounds of carbohydrates in 10 lb. of:

57. Rice (73%). **58.** Bread (47%). **59.** Sugar (98%).

60. Beans (54%). **61.** Oatmeal (65%).

Find the number of pounds of fats in 10 lb. of:

62. Eggs. **63.** Butter (78%). **64.** Milk. **65.** Oatmeal.

66. The heat units in 1 lb. of round of beef are what per cent of the heat units in 1 lb. of butter?

67. The heat units in 1 lb. of smoked ham are what per cent of the heat units in 1 lb. of loin of pork?

68. The heat units in 1 lb. of unskimmed milk are what per cent of the heat units in 1 lb. of eggs?

69. Make similar problems based on this table.

Farm Problems

70. The world's record for the production of butter fat is held by a Holstein, Banostine Belle de Kol. In one year she produced 27404.4 pounds of milk and 1058.34 pounds of butter fat. What was the average per cent of butter fat?

The Ohio Experiment Station made a study of the losses of manure when exposed in flat piles in a barnyard for a period of 3 months. They obtained the following data. The numbers in each case indicate the pounds of the material in a ton of manure. Find the loss per cent of each constituent.

	Organic Matter	Nitrogen	Phosphoric Acid	Potash
January	**71.** 416.00	**72.** 13.66	**73.** 4.90	**74.** 16.28
April	254.79	8.32	3.73	6.29

75. Mr. Fox put 15 dozen eggs, costing \$.35 per dozen, in an incubator. 80 % of the eggs hatched. What was his profit, if he sold the chicks at 8 cents each?

76. What would have been the per cent of profit, if Mr. Fox had raised the chickens at a cost of \$.40 each, and sold them at an average of \$.60 each?

77. Butter bought for 20 ¢ a pound is sold for 35 ¢ a pound. Allowing 5 % for transportation, what is the gain per cent?

78. A farmer sowed $1\frac{1}{2}$ bu. of oats and harvested 60 bu. What per cent of the seed oats was the yield?

79. At an egg-laying contest, the first place was won by a white Leghorn hen from England that laid 282 eggs, and the second place by a white Leghorn hen from Connecticut that laid 269 eggs. What per cent more eggs did the first hen lay than the second?

80. If the average in this contest was 156 eggs, by what per cent did the first hen exceed the average? the second hen?

81. A farmer was offered two samples of cottonseed meal, the first of which, containing 42% of protein, cost $27 per ton, and the second, containing 37%, cost $24 per ton. Making no allowance for other ingredients, how much, to the nearest hundredth cent, did the protein cost per pound in each sample?

82. John picked $3\frac{1}{2}$ pt. of berries, Charles 4 pt., and James 3 pt. What per cent of the whole amount did each pick?

83. A man rented a field to a tenant in return for $33\frac{1}{3}$% of the grain to be raised. The owner of the field sold his share of the grain for 80¢ a bushel, receiving $240. How many bushels did he have, and how many bushels did the tenant have?

84. Mr. Brown planted 8 acres of his land in celery, setting out 25,000 plants to the acre. The yield was 2000 dozen plants to the acre. What per cent of the plants matured?

85. A garden plot 180 feet square is laid out as follows: $16\frac{2}{3}$% of it in potatoes, 20% of the remainder in asparagus, and the remainder in other vegetables. What per cent of the whole field is planted in other vegetables?

At the Buffalo Exposition a six months' dairy cow contest was run with the following results. What was the number of pounds of butter fat produced by each animal? What animals were above the average in production of milk? of butter fat?

	Breed	Pounds of Milk per Cow	Average Test
86.	Guernsey	5425.5	4.60%
87.	Jersey	5397.4	4.52%
88.	Ayrshire	6599.6	3.60%

	Breed	Pounds of Milk per Cow	Average Test
89.	Holstein	7852.0	3.25 %
90.	Red Poll	5738.8	3.98 %
91.	Brown Swiss	6178.4	3.98 %
92.	French-Canadian	4932.8	3.99 %
93.	Shorthorn	6377.1	3.57 %
94.	Polled Jersey	4065.7	4.66 %
95.	Dutch Billed	4978.7	3.40 %

A record was kept of a Guernsey cow, Sweet Brier, at the Minnesota Station for eight years with the following results. Find the number of pounds of butter fat produced each year. What was the average number of pounds of milk produced per year? of butter fat?

	Year	Milk	Per Cent of Butter Fat		Year	Milk	Per Cent of Butter Fat
96.	1st	7057.0	5.13	**100.**	5th	6364.6	5.00
97.	2d	7094.5	5.04	**101.**	6th	7594.5	5.35
98.	3d	4744.9	4.89	**102.**	7th	4258.9	5.04
99.	4th	8426.7	4.98	**103.**	8th	4897.2	4.84

104. A farmer raised 540 bu. of oats. He kept 25 % of them and sold the remainder at $1.25 per bushel. How much did he receive for the part sold? How many bushels did he keep?

105. A farmer who had 1000 doz. eggs for sale which cost him $.21 a dozen, lost 10 % of them by breakage. If he sold the rest at $.30 a dozen, what per cent did he gain?

106. If you buy a farmhouse for $9500 and pay $500 for repairs, for how much per month must you rent it to gain 5 % on your investment, after paying 1 % for taxes?

Suggestion. 6 % of the investment will be the yearly rental.

107. If a farmer sold cattle and horses for $9550, thereby gaining 25 %, how much would he have gained by selling them at a profit of 35 %?

SIMPLE INTEREST

If a man rents or hires a piece of land, he agrees to pay for its use a certain sum per year, called rent. If he borrows or hires money, he agrees to pay for its use a certain *per cent* of it for one year.

A borrows $250 from B for one year, and agrees to pay B the $250 at the end of the year, and also a sum equal to 6% of the $250 borrowed. If A borrows the money for more than one year, he pays for its use 6% of it each year, and a fraction of 6% of it for any fraction of a year. He may pay 5% a year, or any other rate that is agreed upon.

The **principal** is the sum of money borrowed.

The **interest** is the sum paid for the use of the principal.

The **rate of interest** is the per cent of the principal paid for its use for *one year*. Thus, the expression, "interest at the rate of 5%," means that the interest for *one year* equals 5% of the principal.

The **amount** is the sum of the principal and the interest.

1. What is meant by interest at 4%? at 5%? 7%? $2\frac{1}{2}$%? $4\frac{1}{2}$%? $3\frac{3}{4}$%?

2. What is the interest on $100 for one year at 6%? at 5%? $3\frac{1}{2}$%? $6\frac{1}{2}$%? 10%?

3. Find the interest on $400 for 1 year at 4%; at 5%; 6%; $4\frac{1}{2}$%.

4. Find the interest on $400 for 2 years at 4%; at 5%; 6%; $4\frac{1}{2}$%.

Study Recitation

1. What is the interest on $250 for one year at the rate of 5%? the amount?

Solution

Interest	$250 × .05 = $12.50
Principal	$250.00
Amount	$262.50

2. C borrows $120 of D for 2 years 6 months at 5%. What is the interest? the amount?

Solution

Principal	$120
Rate	.05
Interest for 1 year	6.00
Number of years	2½
Interest for ½ year, or 6 months	3.00
Interest for 2 years	12.00
Interest for 2 years 6 months	15.00
Principal	120.00
Amount due at end of 2 years 6 months . .	$135.00

Written

Find the interest and the amount for the given rate and time:

3. $500 for 3 years at 4%.
4. $175 for 3 years at 7%.
5. $125 for 2 years at 6%.
6. $800 for 6 years at 5%.
7. $280 for 1 year 6 months at 6%.
8. $560 for 4 years 4 months at 8%.
9. $600 for 4 months at 6%.
10. $150.50 for 3 years 2 months at 6%.
11. $240 for 1 year 1 month at 6%.
12. $240 for 1 year 5 months at 6%.

In computing interest, 30 days are usually regarded as a month. Since there are 30 days in 1 month, 3 days equal $\frac{1}{10}$ of a month.

The interest for 1 month is $\frac{1}{12}$ of the interest for 1 year. The interest for any number of months is the interest for 1 month multiplied by the number of months.

Any number of months and days may be expressed as months and tenths of a month.

To reduce any number of days to a fraction of a month.

Divide the number of days by 3 *and call the result tenths of a month.*

1. 3 days = .1 of a month.
2. 6 days = .2 of a month.
3. 12 days = .4 of a month.
4. 1 day = .0$\frac{1}{3}$ of a month.
5. 2 days = .0$\frac{2}{3}$ of a month.
6. 5 days = .1$\frac{2}{3}$ of a month.
7. 10 days = .3$\frac{1}{3}$, or $\frac{1}{3}$ mo.
8. 26 days = .8$\frac{2}{3}$ of a month.
9. 17 days = ? of a month.
10. 29 days = ? of a month.
11. 9 days = ? of a month.
12. 18 days = ? of a month.
13. 11 days = ? of a month.
14. 28 days = ? of a month.
15. 20 days = ? of a month.
16. 23 days = ? of a month.
17. 6 months and 6 days = 6.2 months.
18. 7 months and 16 days = 7.5$\frac{1}{3}$ months.
19. 5 months and 13 days = ? months.
20. 4 months and 14 days = ? months
21. 3 months and 20 days = ? months.
22. 4 years, 3 months, and 7 days = 4 years 3.2$\frac{1}{3}$ months.
23. 6 years, 7 months, and 13 days = 6 years 7.4$\frac{1}{3}$ months.
24. 9 months and 18 days = 9.6 months.
25. 11 months and 23 days = ? months.
26. 3 months and 17 days = ? months.

Study Recitation

1. What is the interest on $150 for 6 yr. 7 mo. 13 da. at 5 % per annum?

Solution

$150
.05
Interest on $150 for 1 yr. at 5 % $7.50
6
Interest on $150 for 6 yr. $45.00
12)$7.50
Interest on $150 for 1 mo. $.625
7 mo. and 13 da. = 7.4⅓ mo. 7.4⅓
2081/3
2500
4375
Interest on $150 for 7.4⅓ mo. $ 4.6458⅓
Interest on $150 for 6 yr. $45.00
Interest on $150 for 6 yr. 7 mo. 13 da. . . $49.65

Note. In final results in interest, count 5 mills or more as an extra cent, and discard less than 5 mills.

2. Find the amount of $250 for 3 yr. 9 mo. 17 da. at 7 %.

Solution

$250
.07
Interest on $250 for 1 yr. at 7 % $17.50
3
Interest on $250 for 3 yr. $52.50
12)$17.50
Interest on $250 for 1 mo. $1.458+
9 mo. 17 da. = 9.5⅔ mo. 9.5⅔
972
7290
13 122
Interest on $250 for 9.5⅔ mo. $ 13.9482+
Interest on $250 for 3 yr. $ 52.50
Interest on $250 for 3 yr. 9 mo. 17 da. . . . $ 66.45
Principal $250.00
Amount of $250 for 3 yr. 9 mo. 17 da. at 7 % $316.45

WRITTEN

Find the interest on:

3. $2500 for 60 days at 5%.
4. $125 for 2 years 9 months at 7%.
5. $900 for 90 days at $6\frac{1}{2}$%.
6. $600 for 2 years 8 months at 5%.
7. $850 for 1 year 10 days at 6%.
8. $625.80 for 8 months 15 days at 5%.

What is the interest and the amount on each of the following at 6%? at 7%? at 8%? at 10%?

9. $180 for 3 years 9 months 18 days.
10. $165 for 2 years 7 months 9 days.
11. $224 for 6 years 2 months 4 days.
12. $168 for 2 years 11 months 6 days.
13. $250 for 2 years 5 months 12 days.
14. $750 for 4 years 6 months.
15. $4200 for 5 months 6 days.
16. $75,000 for 1 year 6 months 15 days.

Find the interest on the following at 6%; at 5%; at 4%; at $4\frac{1}{2}$%; at 8%; at 3%.

17. $423 for 5 mo.
18. $189 for 3 mo.
19. $686.50 for 8 mo.
20. $87.75 for 3 yr. 6 mo.
21. $457 for 2 yr. 4 mo.
22. $595 for 15 mo.
23. $638 for 10 mo.
24. $742 for 14 mo.
25. $844 for 9 mo.
26. $963 for 15 mo.
27. $738 for 12 mo.
28. $609 for 6 yr. 5 mo.
29. $384 for 1 yr. 8 mo.
30. $489.60 for 2 yr. 3 mo.
31. $514 for 1 yr. 5 mo.
32. $840 for 1 yr. 7 mo.

33. $892 for 2 yr. 3 mo.
34. $576 for 5 yr. 6 mo.
35. $731 for 3 yr. 5 mo.
36. $816 for 2 yr. 2 mo.
37. $987 for 5 mo.
38. $3490 for 6 yr. 2 mo.
39. $996 for 7 mo.
40. $684 for 4½ mo.
41. $377 for 3 yr. 1 mo.
42. $468 for 15 da.
43. $559.50 for 5 yr. 8 mo.
44. $743 for 4 yr. 3 mo.
45. $634 for 2 yr. 2 mo.
46. $224 for 8 yr. 5 mo.
47. $313.25 for 2 yr. 2 mo.
48. $428.50 for 6 yr. 4 mo.
49. $530 for 60 days.
50. $748 for 90 days.
51. $857 for 30 days.
52. $9650 for 5 mo.
53. $985.40 for 7 mo.
54. $89 for 8 mo. 10 days.
55. $486 for 6 mo. 6 days.
56. $576.30 for 3 yr. 4 mo.
57. $636.90 for 1 yr. 6 days.
58. $624 for 1 yr. 9 mo. 6 days.
59. $413 for 1 yr. 3 mo. 15 days.
60. $385 for 1 yr. 16 days.

SIX PER CENT METHOD

STUDY RECITATION

Another method of computing interest, called the 6 % method, is frequently employed.

The interest on $1 for 1 year at 6 % = 6 cents.
The interest on $1 for 2 months (⅙ of a year) = 1 cent.
The interest on $1 for 1 month, or 30 days = ½ of 1 cent, or 5 mills.
The interest on $1 for 6 days (⅕ of 30 days) = 1 mill.
The interest on $1 for 1 day = ⅙ of a mill.

1. Find by the 6 % method the interest on $200 for 2 yr. 6 mo. 6 days at 6 %.

SOLUTION

Interest on $1 for 2 yr. at 6 %	$.12
Interest on $1 for 6 mo. at 6 %	.03
Interest on $1 for 6 da. at 6 %	.001
Interest on $1 for 2 yr. 6 mo. 6 da. at 6 %	$.151

The interest on $200 = 200 × $.151, or $30.20.

2. Find the interest on $500 for 60 days at 6 %.

SOLUTION

Since the interest at 6 % for 60 days is 1 % of the principal, it may be found by moving the decimal point two places to the left. 1 % of $500 = $5.

3. What part of 60 days are 30 days? 10 days? 6 days? 120 days or 2 mo.? 90 days or 3 mo.?

4. Since the interest of $100 for 60 days is $1, what is the interest of $100 for 30 days? for 120 days?

To find the interest on $1 for any period at 6 %, take 6 times as many cents as there are years plus $\frac{1}{2}$ as many cents as there are months plus $\frac{1}{6}$ as many mills as there are days.

To find the interest on any sum, for any time, at 6 %, multiply the interest on $1, for the given time, at 6 %, by the number of dollars.

To find the interest on any sum for 60 days at 6 %, move the decimal point two places to the left.

What is the interest on $1 for the following periods of time at 6 % :

5.	2 years?	8 months?	2 years	8 months?	
6.	8 years?	10 months?	8 years	10 months?	
7.	3 years?	3 months?	3 years	3 months?	
8.	9 years?	7 months?	9 years	7 months?	
9.	10 years?	11 months?	10 years	11 months?	
10.	18 days?	15 days?	13 days?	23 days?	2 days?
11.	24 days?	21 days?	14 days?	26 days?	5 days?
12.	12 days?	27 days?	16 days?	29 days?	4 days?
13.	1 day?	7 days?	17 days?	11 days?	6 days?
14.	9 days?	8 days?	19 days?	3 days?	22 days?
15.	10 days?	25 days?	60 days?	30 days?	120 days?

16. 4 months 12 days? 5 months 7 days?
17. 7 months 18 days? 3 months 21 days?
18. 9 months 24 days? 8 months 23 days?
19. 10 months 6 days? 7 months 25 days?
20. 10 months 9 days? 9 months 16 days?
21. 2 years 6 months 12 days? 4 years 11 months?
22. 3 years 5 months 18 days? 12 years 7 months 3 days?
23. 4 years 7 months 24 days? 5 years 8 months?
24. 1 year 6 months 7 days? 7 years 9 months 15 days?
25. 5 years 8 months 27 days? 8 years 11 months?

Written

Find the interest on each of the following principals, for the given time, at 6%:

26. $250 for 3 yr. 6 mo. 18 days.
27. $175 for 2 yr. 9 mo. 24 days.
28. $215 for 4 yr. 10 mo. 12 days.
29. $125 for 6 yr. 6 mo. 9 days.
30. $120.75 for 3 yr. 7 mo. 18 days.
31. $118.50 for 4 yr. 8 mo. 15 days.
32. $500 for 8 yr. 2 mo. 4 days.
33. $625 for 1 yr. 1 mo. 1 day.
34. $840 for 1 yr. 6 mo. 15 days.
35. $920 for 2 yr. 4 mo. 20 days.
36. $750 for 3 yr. 3 mo. 10 days.
37. $320 for 4 yr. 9 mo. 6 days.
38. $1500 for 3 yr. 8 mo. 5 days.
39. $1600 for 2 yr. 2 mo. 18 days.
40. $1750 for 1 yr. 5 mo. 15 days.

41. $940.25 for 1 yr. 7 mo. 10 days.

42. $450.62 for 2 yr. 10 mo. 6 days.

43. $830.91 for 3 yr. 11 mo. 5 days.

To find the interest on any sum, for any time, at any rate per cent.

STUDY RECITATION

1. The interest on a given sum of money, for a given time, at 6%, is $24. Find the interest on the same sum, for the same time, at 1%; at 5%; at 7%.

SOLUTION. Interest on the same sum for the same time, at 1%, is ⅙ of $24, or $4. At 5% it is 5 × $4; at 7%, 7 × $4.

From the interest on any principal at 6%, the interest at other rates may be found by adding or subtracting aliquot parts of the interest at 6%, as follows:

Adding	⅙	of itself for the interest at 7%
Subtracting	⅙	of itself for the interest at 5%
Adding	⅓	of itself for the interest at 8%
Subtracting	⅓	of itself for the interest at 4%
Dividing by	4	for the interest at 1½%
Adding	¼	of itself for the interest at 7½%
Subtracting	¼	of itself for the interest at 4½%
Dividing by	2	for the interest at 3%

WRITTEN

2–127. Solve examples 26 to 43 (pp. 344, 345), substituting rates of 3%, 4%, 4½%, 5%, 2½%, 5½%, 3¾%.

128. Mr. H. H. Jones borrows $1200 June 5, and pays the debt October 23. What is the interest at 4½%?

129. Find the interest on a debt of $900 from Jan. 5, 1914, to Dec. 30, 1914 at 5%.

130. Find the amount due on a debt of $360 from July 7, 1913, to Jan. 20, 1914 at 6%.

EXACT INTEREST

Exact interest, or **accurate interest**, as it is sometimes called, is found by taking the *exact* number of days between two dates, and reckoning 365 days to a common year, and 366 days to a leap year.

Interest is usually reckoned on the basis of 12 months of 30 days each, or 360 days to the year. If the time for which the principal is on interest is stated in years, or in years and months, the usual method also gives the exact interest; but if the time the principal is on interest is given in days, the usual method does not give the exact interest. If the time the principal is on interest is less than 1 year, and if the dates between which the principal draws interest are given, the exact number of days may be found, but the usual method will not give the exact interest.

The interest on $1460 for 60 days, at 6%, is found by the usual method to be $14.60.

This is upon the supposition that 60 days are $\frac{60}{360}$ of a year. But 360 days are exactly $\frac{360}{365}$, or $\frac{72}{73}$, of a common year; and $\frac{360}{366}$, or $\frac{60}{61}$, of a leap year. If a man pays the interest for the use of money for 360 days, when he should pay for the use of the money for 365 days or for 366 days, he pays $\frac{5}{365}$, or $\frac{1}{73}$, too much in the common year, and $\frac{6}{366}$, or $\frac{1}{61}$, too much in the leap year.

To find exact interest when the time is in days, diminish the interest as found in the usual way by $\frac{1}{73}$ of itself if the time is a part of a common year; diminish it by $\frac{1}{61}$ of itself if the time is a part of a leap year.

If the time is given in years, months, and days, find the interest for the years and months by the usual method, and to it add the exact interest for the days.

1. Find by the 6% method the interest on $7300 for 60 days at 6%.

2. Find the exact interest, in a common year, on $7300 for 60 days at 6%. By how much does this differ from the answer in example 1?

To find the exact number of days between two dates.

1. How many days are there between May 20 and November 15 of the same year?

May 20 to 31 =	11 days
June	30 days
July	31 days
August	31 days
September	30 days
October	31 days
November	15 days
	179 days

To the number of days remaining in the earliest month given, add the number indicated by the date of the latest month given, and also the number of days in each of the intervening months.

The number of days remaining in May is 11. The latest month named is November, and the date given is 15. The number of days in each of the intervening months, respectively, is 30, 31, 31, 30, 31. The sum of all is 179.

2. Find the time from June 15, 1910 to Sept. 20, 1914.

From June 15, 1910 to June 15, 1914, it is 4 years.

From June 15, 1914 to Sept. 20, 1914, it is 15 days + 31 days + 31 days + 20 days, or 97 days.

The entire time is 4 years 97 days; counting 1 leap year, it is 4 years 98 days.

WRITTEN

3. Find the time from Feb. 1, 1912 to Sept. 4, 1912.

NOTE. Since 1912 was a leap year, count 29 days in February.

4. Find the time from April 3, 1913 to Jan. 5, 1914.

5. Find the exact interest at 6 % on $1200 from Jan. 1, 1913 to June 28, 1913.

The following table is often used for convenience in reckoning time between two dates:

TABLE OF DAYS BETWEEN DATES

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan. . .	365	31	59	90	120	151	181	212	243	273	304	334
Feb. . .	334	365	28	59	89	120	150	181	212	242	273	303
Mar. . .	306	337	365	31	61	92	122	153	184	214	245	275
Apr. . .	275	306	334	365	30	61	91	122	153	183	214	244
May . .	245	276	304	335	365	31	61	92	123	153	184	214
June . .	214	245	273	304	333	365	30	61	92	122	153	183
July . .	184	215	243	274	304	335	365	31	62	92	123	153
Aug. . .	153	184	212	243	273	304	334	365	31	61	92	122
Sept. . .	122	153	181	212	242	273	303	334	365	30	61	91
Oct. . .	92	123	151	182	212	243	273	304	335	365	31	61
Nov. . .	61	92	120	151	181	212	242	273	304	334	365	30
Dec. . .	31	62	90	121	151	182	212	243	274	304	335	365

If you want to find the number of days from May 4 to Sept. 4, follow the horizontal line beginning with May in the left-hand column until it crosses the column headed September. The number at the crossing is 123 days. The number of days from May 4 to Sept. 8 would be 4 more, or 127 days.

(If February 29 for leap year comes between the dates, add 1 day.)

PROBLEMS

WRITTEN

Find the exact interest due on the following:

1. $75 for 36 days at 6%.
2. $125 for 66 days at 6 %.
3. $250 for 24 days at 8 %.
4. $325 for 33 days at 7 %.
5. $115.25 for 80 days at 5%; at 6%.
6. $1000 for 90 days at 4%; at 3%.
7. $3500 for 30 days at 3%; at $4\frac{1}{2}$%.
8. $275 from Jan. 12, 1914 to May 13, 1914, at 6 %.
9. $315 from Sept. 24 to Dec. 28 of the same year at 7 %.
10. $425 from April 20, 1911 to Nov. 12, 1914, at 4 %.

PROBLEMS IN INTEREST

STUDY RECITATION

Given the principal, the interest, and the time, to find the rate.

1. At what rate of interest will $100 yield $12 in 2 yr. ?

SOLUTION. The interest of $100 at 1 % for 2 years is $2. If $100 in 2 years yields $12, the rate per cent equals $12 ÷ $2, or 6. *Ans.* 6 %.

Divide the given interest by the interest for the given time at 1 *%.*

WRITTEN

2. At what rate will $200 yield $36 in 3 years?

3. At what rate will $426 yield $53.25 in 2 years 6 months?

4. The interest of $560 for 3 years 3 months is $145.60. What is the rate?

5. A principal of $260 yields $29.90 in 2 years 3 months 18 days. What is the rate?

6. At what rate will a principal double itself in 10 years? in 20 years? in 25 years?

SUGGESTION. 100 % ÷ 10 = 10 %.

Given the principal, the interest, and the rate, to find the time.

STUDY RECITATION

1. In what time will $100 gain $12 interest at 6 %?

SOLUTION. The interest on $100 at 6 % for 1 year is $6. The number of years is $12 ÷ $6, or 2. *Ans.* 2 yr.

Divide the given interest by the interest at the given rate for 1 *year.*

WRITTEN

2. In how many years will $600 gain $120 at 5%?

3. In how many years will $700 gain $56 at 4%?

4. In how many years will $800 gain $72 at 3%?

5. In how many years will a principal double itself (gain 100% of itself) at 5%?

SOLUTION. 100% ÷ 5% = 20. *Ans.* 20 years.

6. In how many years will a principal double itself at 6%? 10%? 4%?

Given the interest, the rate, and the time, to find the principal.

STUDY RECITATION

1. What principal will yield $12 in 2 years at 6%?

SOLUTION. Since $1 in 2 years at 6% yields $.12, as many dollars must be invested as $.12 is contained times in $12. $12 ÷ $.12 = 100. Hence, $100 is the principal.

Divide the given interest by the interest on $1 *for the given time at the given rate.*

WRITTEN

2. What principal will yield an interest of $600 in 5 yr. at 6%?

3. What principal will yield an interest of $300 in 2 yr. at 5%?

4. What principal will yield an interest of $480 in 4 yr. at 4%?

5. What principal will yield an interest of $50.40 in 3 yr. at 3%?

6. What principal will yield an interest of $69.30 in 2 yr. at $4\frac{1}{2}$%?

PROMISSORY NOTES

A **promissory note** is a written promise made by one person to pay to another a specified sum of money on demand or at a definite time.

PROMISSORY NOTE

$ 600 Fort Wayne, Ind., June 13, 1914.

Three months after date I promise to pay to the order of Charles Rice

Six hundred and $\frac{00}{100}$ Dollars.

Value received

James Wilson.

The **maker** of the note is the one who signs it; that is, the one who is to pay it.

The **payee** is the person to whom it is payable.

The **face** of a note is the sum named in it.

Essentials of a Promissory Note:

1. It must be signed by the **maker.**
2. It should promise to pay to the **payee** or to his **order.**
3. It should promise to pay a **definite sum of money,** the **face.**
4. It should state the **place** where and the **time** when it is given.
5. It should state **when** the money is to be paid.
6. It should state **with interest** and the **rate,** if it is an interest-bearing **note.**

The words "for value received" are usually included in a note, but are not essential.

Who is the maker of the note on p. 351? the payee? What is the face of the note? When is it dated? When is it due?

The note on p. 351 is a **time note** because it is payable at a specified time. A **demand note** is one which the maker may be called upon to pay at any time after date. If in the above note you substitute the words "on demand" for "three months after date," it will become a demand note.

A note is said to **mature** at the expiration of the time specified in the note. A note is due at **maturity.**

A **negotiable note** is a note made payable to the "bearer" or to the "order" of the payee. A note is non-negotiable when it is made payable to the payee only. A negotiable note may be transferred from one person to another.

NOTE. In a few states 3 days of grace are allowed after the expiration of the time mentioned in the note. Days of grace are not to be considered in the examples in this book.

STUDY RECITATION

1. $500.00 INDIANAPOLIS, IND., Aug. 14, 1914.

Six months after date, for value received, I promise to pay to J. C. Root, or order, Five Hundred Dollars, with interest at the rate of 6 per cent per annum.

J. H. JOHNSON.

2. $200.00 MILWAUKEE, WIS., Sept. 15, 1914

Thirty days after date, for value received, I promise to pay to John Jones, or bearer, Two Hundred Dollars, with interest at 7 %.

HENRY SMITH.

Both of the above notes are *negotiable.* What words should be omitted from each note to make it *non-negotiable?*

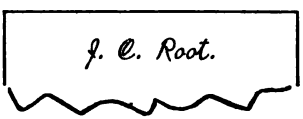
Name the *maker*, the *payee*, and the *holder* of each note.

What is the *face* of each note? What is the date of its *maturity?*

The **indorser** is the person who writes his name on the back of a note to transfer it, if it is made payable to his order, or to guarantee its payment in case the maker fails to pay it when due.

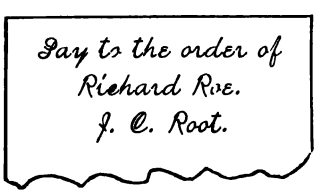
If the holder of the first note on p. 352 wishes to transfer it to John Doe, he must indorse it; that is, write his name on the back of the note. If he writes his name "J. C. Root" and nothing else on the back of the note, it is an **indorsement "in blank,"** and the note is then payable to the holder. The indorsement by J. C. Root renders him liable for the payment in case J. H. Johnson is unable to pay the note.

INDORSEMENT IN BLANK



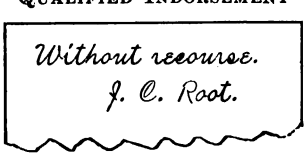
If Mr. Root writes on the back of the note "Pay to the order of Richard Roe, J. C. Root," it is an **indorsement "in full,"** and the note is then payable to Richard Roe, or to the person to whom he may transfer it by his indorsement. The indorser is liable for payment of the note in case the maker fails to pay it.

INDORSEMENT IN FULL



If Mr. Root writes his name on the back of the note, as in either of the above cases, and adds the words "without recourse," it is an **indorsement without recourse,** and the indorser is not liable for payment of the note in case the maker fails to pay it.

QUALIFIED INDORSEMENT



DIFFERENT FORMS OF NOTES

3. $100.25 TERRE HAUTE, IND., June 1, 1914.

Sixty days after date I promise to pay John Jones, or order, One Hundred and $\frac{25}{100}$ dollars, with interest, value received.

A. J. HENRY.

When a note is drawn "with interest," but with no rate specified, the legal rate of the state in which the note is drawn is understood.

4. $150.00 CLEVELAND, O., April 4, 1913.

On demand, for value received, I promise to pay A. H. George, or order, One Hundred Fifty Dollars, with interest at 6%.

A. B. SMITH.

When is the above note due?

If a note makes no mention of interest, no interest can be collected until it matures. If the note is not paid at maturity, it draws legal interest from the date of maturity to the date of payment.

5. $150.00 MILWAUKEE, WIS., May 5, 1914.

Three months from date, I promise to pay John Jones, or order, One Hundred Fifty Dollars, value received.

JAMES JOHNSON.

When did this note mature?

It was not paid until Dec. 5, 1914. For what length of time did it draw interest? why? at what rate? What amount was required to pay the note?

6. $100.00 MADISON, WIS., Feb. 12, 1914.

Six months after date, for value received, we jointly and severally promise to pay James Smith, or order, One Hundred Dollars, with interest at 7%.

JOHN DOE,
RICHARD ROE.

This note is called a **joint and several note**, and its makers, John Doe and Richard Roe, are jointly and singly liable for its payment. If the words "and severally" were omitted, the note would be a **joint note**, and each of the makers would be liable for payment of only one half the amount due at maturity.

A **protest** is a written notice, stating that the maker has failed to pay the note. It is sent to the indorsers by a notary public, in a manner prescribed by law.

A note maturing on a Sunday or on a legal holiday is made payable by law in most states the day before it matures.

7. Write a negotiable note for $750, with interest at 6 %, making yourself the payee and your teacher the maker. Find the amount due at maturity.

8. Write a negotiable note for $175.25, payable on demand, with interest at 7 %, making one of your classmates the payee and yourself the maker.

Have the payee fix the time of payment. Find the amount due.

9. Write a negotiable demand note for $250, without interest, making yourself the payee and John Smith the maker.

Fix the time when payment is demanded. If the note is not paid until 6 months after demand, what amount is due at the time of payment?

10. Write a negotiable note for $550.25, with interest at 8 %, making yourself the payee and your teacher the maker. Indorse it in blank for transfer. What amount is due at maturity?

11. Write a note from the following data: Negotiable; face, $2500; maker, J. Smith; payee, yourself; maturity, 1 year 6 months from date; interest, 5 %. Find the amount due at maturity.

12. A 60-day note for $550, with simple interest at 10%, dated March 1, 1914, was paid at maturity. When was the note due? What was the amount due?

13. A 30-day note for $1000, without interest, was paid three months after it matured. What was the amount due, the legal rate being 6%?

14. A 60-day note for $875, with interest at 6%, was paid in ninety days. What was the amount paid, reckoning simple interest? exact interest?

15. A two-months' note for $1500, with interest at 5%, dated April 1, 1914, was paid at maturity. Find the date of maturity and the amount due.

NOTE. For partial payments on notes, see Supplement, pp. 483, 484.

MORTGAGES

A **mortgage** is a conveyance of real estate or some interest therein, as security for a loan.

The **mortgagor** is the person who borrows money and gives the mortgage on the property; the **mortgagee** is the person who lends the money on the property as security.

1. Mr. Bent sells a house to Mr. Gordon for $12,000, receiving $7000 cash and a mortgage on the house for $5000 at 5%. What yearly interest must Mr. Gordon pay Mr. Bent on this mortgage?

SOLUTION. 5% of $5000 = $250.

2. Mr. Allen invests $6500 in a guaranteed mortgage at $4\frac{1}{2}$%. How much yearly interest does Mr. Allen receive on this mortgage?

3. Find the interest on a mortgage of $25,000 at $5\frac{1}{2}$%.

4. Find the semiannual interest on a mortgage for $12,250 at 5%.

BANKS AND BANKING

A **bank** is an establishment that receives, lends, exchanges, or issues money.

National banks are under the control of the government, and state banks under the control of the state. There are also many private banks.

Federal reserve banks are central banks in the national banking system established for the purpose of enlarging our currency system and lending money to other banks when needed.

Trust companies are also permitted in many states to do a general banking business, though they exercise many other powers as well.

CHECKS

A **check** is a written order by a depositor in a bank, directing the payment of a specified sum of money.

STUB	CHECK
No. 3586 Date May 1, '14 Payable to Frank Arnold For Mdse. Am't. $500.25	South Bend, Ind., May 1, 1914, No. 3586 First National Bank Pay to the order of Frank Arnold $500.25 Five hundred and 25/100 ~~~~~~~~~~ Dollars. L. A. Clarke.

The terms **maker**, **payee**, **face**, **negotiable**, etc., apply to checks as well as to promissory notes. The various forms of indorsement are also applied in the same manner. The stub that remains in the check book after the check is torn out contains a record of the check.

DEPOSIT SLIP

DEPOSITED BY

Charles Spencer

IN THE

First National Bank

South Bend, Ind., Jan. 4, 1914.

Bills	$ *15*	
Gold	*10*	
Silver	*13*	*50*
Check on Lee. Nat. Bank	*65*	*14*
" " Am. Exch. "	*14*	*33*
Total	*117*	*97*

If you wish to open an account, you take your money to the bank, and give it with a slip filled out like the one here shown to the "cashier" or "receiving teller." You then receive a book in which the amount is credited to you, and another book of blank checks.

If you wish to draw money for yourself from a bank, you may write a check payable to "cash" which need not be indorsed or a check payable to "self" which must be indorsed.

A check payable "to bearer" may be collected by any one presenting it.

WRITTEN

1. A man's balance at the Union Trust Co. was $2314.57. Find his balance after checking out $15.75, $14.83, $72.94, and $83.57. Write the checks and the stubs, choosing your own payees.

2. A lady's balance at the Second National Bank was $653.75 on June 1. On June 3 she drew a check for $54.65 payable to Arthur Hart, and on June 15 she deposited $140

in bills. What was her balance July 1? Write the check, the stub, and the deposit slip.

3. Draw checks, filling in items as you choose, (1) payable to cash; (2) payable to bearer; (3) payable to self; (4) payable to Frank Abbott. Indorse the checks when necessary.

4. George Parker's balance at the bank Jan. 1 was $5000. He deposited on Jan. 18, two $50 bills and checks for $500.75, $625.44, $173.98, $99.58, $44.67, and $74. He drew checks during the month for $264, $783.95, $42.67, and $83.54. What was his balance Feb. 1? Write the checks, the stubs, and the deposit slips, choosing banks and payees.

5. A lady's balance in the bank at the beginning of a month was $584.50. After she had deposited $135.40, $654.78, and $829.40 and drawn checks for $45.64, $15.32, and $1.75, what was her balance? Write the checks, the stubs, and the deposit slips, supplying names, banks, and other items.

6. John Farmer's balance at the Corn Exchange Bank on Feb. 1, 1914 was $1213.56. On Feb. 3 he deposited a check for $145.16, $14.50 in silver, and two $5 gold pieces. On Feb. 4 he drew a check to the order of Thomas Arnold for $19.56. Find his balance on Feb. 5. Write the checks, the stubs, and the deposit slip.

7. Harold Aitken's balance at the New York Produce Exchange Bank on April 1, 1914 was $650.69. On April 2, he drew $17.45; on April 3, $16.50. On April 4, he deposited a check for $110.19 and bills for $17.50. On April 19, he drew a check for $25.60; and on April 30, a check for $17.40. Find his balance on May 1, 1914. Write the checks, the stubs, and the deposit slip.

BANK DISCOUNT

STUDY RECITATION

Bank discount is a sum charged by a bank for payment of a note or a draft before it becomes due.

This sum is the interest, at the bank's rate of discount, on the *amount* of the note or the draft at maturity, from the date the note is discounted to the date it matures.

When a bank lends money to the borrower on his note and takes bank discount, it deducts the interest from the face of the note at the time the note is given, and pays the maker of the note the difference. This difference between the face of the note and the interest is called the *proceeds* of the note.

If A borrows $100 from B and gives B a note for that sum, due in 4 months at 6%, A receives $100, and *4 months afterward* pays B $100 plus the interest on $100 for 4 months at 6%, or $2, making $102. If, however, A gives his note for $100, due in 4 months, to a bank, and the bank discounts it at 6%, the interest on the face of the note for 4 months at 6%, or $2, is *at once* deducted from the face of the note and retained by the bank, and only $98 is paid to A, the maker of the note. This is called the **proceeds** of the note. At maturity, A, the maker of the note, pays the bank $100. The note given to the bank does not specify interest until it matures. If the note is not paid at maturity, it then begins to draw interest. In the first case, A, the maker of the note, receives from B the amount specified in the note, and pays it with interest at maturity. In the second case, the interest on the amount of the note is paid by A in advance to the bank, and the maker, A, receives from the bank the proceeds, or the amount of the note less the interest.

A bank not only discounts notes made by its customers direct to the bank, but it also buys or discounts notes owned by its customers and payable to them. In such cases, the owner of the note indorses it, making it payable to the bank. Such notes may or may not bear interest.

When such a note does not bear interest, the holder receives the face of the note less discount on it, at the bank's rate, from the date it is discounted to the date of maturity.

The **proceeds** of a note is the sum due at maturity less the bank discount.

The **term of discount** is the time from the date of discount to the date of maturity.

1. $100.00 VINCENNES, IND., Mar. 3, 1914.

Three months from date, for value received, I promise to pay John Adams, or order, One Hundred Dollars.

PETER SMITH.

This note was due 3 months after March 3, 1914, or June 3, 1914; but on April 3 (**date of discount**), the payee, John Adams, needing money, took the note to the First National Bank to have it discounted. He indorsed the note, making it payable to the cashier of the bank. The bank, being asked to cash the note two months before it became due, charged interest on it at its rate of discount, 6%, for the two months. The interest was $1. This sum was deducted from the $100, the amount payable to the bank at **maturity** (two months later), by Peter Smith. John Adams received the **proceeds**, $99.

The transaction was the same as if John Adams had given his own note for $100 to the bank April 3, payable in two months, without interest, and the bank had discounted it.

If to the above note the words "with interest at 6%" were added, the amount due at maturity would be $100 plus $1.50 (the interest for 3 months at 6%), or $101.50. If John Adams had the note discounted April 3, as above, the bank would compute interest at its rate of discount, as before, on the amount due at maturity, $101.50. The discount on $101.50 at 6% for 2 months, the time from the date the note is discounted to maturity, is $1.02. This is the bank discount. John Adams would receive the proceeds, $101.50 − $1.02, or $100.48.

The bank's rate of discount may be different from the rate of interest specified in the note.

2. $500.00 MILWAUKEE, WIS., March 10, 1914.

Ninety days after date, for value received, I promise to pay to the order of Richard Roe, Five Hundred Dollars, with interest at 6%.

JOHN DOE.

This note was discounted at a bank, April 3. The bank's rate of discount was 5%. What were the proceeds?

SOLUTION. This note at maturity, June 8, 1914, would yield the bank $500 plus the interest on that sum for 90 days at 6%. The interest is $7.50. $500 + $7.50 = $507.50.

As the note was brought to the bank for discount on April 3, it had 66 days yet to run to June 8. The bank computed discount on the amount of the note, $507.50, for 66 days at 5%. The discount was $4.65. The proceeds of the note were $507.50 − $4.65, or $502.85.

WRITTEN

Find the bank discount and the proceeds of a note for:

3. $200, payable in 60 days, discounted at 6%.
4. $125, payable in 90 days, discounted at 7%.
5. $175, payable in 30 days, discounted at 8%.
6. $250, payable in 3 months, discounted at 9%.
7. $150, payable in 45 days, discounted at 6%.
8. $1000, payable in 30 days, discounted at 5%.
9. $1500, payable in 60 days, discounted at $4\frac{1}{2}$%.
10. $1600, payable in 90 days, discounted at 6%.
11. $2500, payable in 45 days, discounted at $5\frac{1}{2}$%.
12. $3000, payable in 2 months, discounted at 6%.
13. A note for $200 was given Jan. 5, 1914, payable in 90 days. When did it mature?
14. A note for $450, bearing 6% interest, was given Nov. 3, 1910, payable in 3 months. When did it mature? It was discounted at a bank, December 10. The rate of discount was 7%. For what length of time was it discounted? What was the discount? What were the proceeds?
15. A note for $600, bearing 6% interest, was given Oct. 5, 1913, payable in 6 months. It was discounted at a bank Jan. 5, 1914, at 5%. Find the date of maturity, the discount, and the proceeds.

Find the discount and proceeds of each of the following interest-bearing notes:

	Face of Note	Date	Time	Rate of Interest	Date of Discount	Rate of Discount
16.	$750	Sept. 10	60 days	6%	Oct. 15	5%
17.	$175	Aug. 12	90 days	7%	Oct. 20	6%
18.	$125	June 20	3 months	8%	July 20	6%
19.	$225	June 1	4 months	6%	Aug. 1	4%
20.	$500	Jan. 15	60 days	4%	Feb. 15	6%
21.	$1000	Jan. 5	30 days	5%	Jan. 10	5%
22.	$2500	March 3	60 days	6%	March 4	6%
23.	$1800	April 7	90 days	4%	April 15	6%
24.	$3500	May 6	45 days	5%	May 15	5%
25.	$6000	July 5	60 days	4%	July 15	6%

National banks are required to keep on reserve certain percentages of their deposits. Demand deposits are those payable within 30 days; time deposits, those payable after 30 days or subject to not less than 30 days' notice.

Country banks must keep on reserve 12% of demand and 5% of time deposits; *reserve city banks*, 15% of demand and 5% of time deposits; and *central reserve city banks*, 18% of demand and 5% of time deposits.

26. Find the amount that must be kept on reserve by a country bank having demand deposits of $310,500 and time deposits of $49,260.

27. Find the amount that must be kept on reserve by a reserve city bank having demand deposits of $50,625,200 and time deposits of $9,635,204.

28. Find the amount that must be kept on reserve by a central reserve city bank having demand deposits of $75,632,410 and time deposits of $21,216,308.

SAVINGS BANK ACCOUNTS

A **savings bank** is a bank, under the control of the government, established for the purpose of receiving deposits of money and paying interest thereon.

Interest is usually paid on any number of dollars from $1 to $3000, on condition that such sum has been on deposit for an entire **interest term**, the period between which payments are due, as January 1 and July 1, in some banks; January 1, April 1, July 1, and October 1 in others; or the first of each month.

Each depositor is provided with a **bank book** in which are entered his deposits and the amounts he draws out.

Most savings banks credit interest at the end of every interest term, on the smallest balance on deposit during the entire term. Usually no interest is allowed on cents. Consult a bank in your neighborhood for its custom in this matter.

When interest is due, it may be withdrawn or it may be placed to the credit of the depositor. In the latter case it draws interest the same as other deposits. Savings banks, therefore, pay compound interest.

Compound interest is interest on the amount of the principal and on the interest, if the interest is unpaid when due.

Study Recitation

1. Find the amount and the compound interest of $1000 for 2 yr. 6 mo. at 6% with interest compounded annually.

Solution

Principal	$1000	
Interest for 1st yr. at 6%	60	
Principal at beginning of 2d yr.	$1060	
Interest for 2d yr. at 6%	63.60	
Principal at beginning of 3d yr.	$1123.60	
Interest for 6 mo. at 6%	33.708	
Amount for 2 yr. 6 mo.	$1157.308, or	$1157.31
Less 1st principal		1000
		$ 157.31, Compound int.

NOTE. If interest is compounded semiannually, take ½ the rate and double the number of periods. Thus, 2 yr. at 6% semiannually is the same as 4 yr. at 3% annually.

WRITTEN

2. What is the amount of $3000 for 2 yr. at 6% compounded annually? the compound interest?

3. What is the amount of $4000 for 2 yr. at 4% compounded semiannually? Compound interest is usually reckoned by tables. See p. 481.

4. The following is a statement of Frank Dow's deposits and withdrawals, with interest compounded semiannually at 4% per annum.

STATEMENT

DATE	DEPOSITED	DRAWN OUT	INTEREST	BALANCE
1913				
Dec. 28	125 00			125 00
1914				
Jan. 1				125 00
March 2		25 00		100 00
July 1			2 00	102 00
July 2	100 00			202 00
1915				
Jan. 1			4 04	206 04

The deposit of Dec. 28, 1913 did not begin to draw interest until January 1, 1914, and the interest for the half year from January 1 to July 1 was computed on the smallest balance for the half year, namely on $100, since $25 was withdrawn before July 1. Money deposited July 2 is allowed to draw interest from July 1, hence the interest due Jan. 1, 1915 is reckoned on $102 + $100, or $202.

NOTE. A fractional part of a cent on the interest is usually dropped.

5. Find the balance due January 1, 1915 on the following, with interest at 4 %, due January 1 and July 1. Arrange the examples as in the form on p. 365.

Deposits		Withdrawals	
Jan. 2, 1914	$1000	April 1, 1914	$ 10
March 1, 1914	200	June 1, 1914	25
July 1, 1914	600		

6. Find the balance due January 1, 1915 on the following account, with interest at $3\frac{1}{2}$ %, due January 1 and July 1.

Deposits		Withdrawals	
July 2, 1914	$500	Oct. 2, 1914	$40
Aug. 1, 1914	50	Oct. 25, 1914	35
Sept. 1, 1914	75		
Oct. 1, 1914	100		

7. Find the balance due January 1, 1915 on the following, with interest at 4 %, due January 1, April 1, July 1, and October 1.

Deposits		Withdrawals	
Jan. 2, 1914	$600	May 1, 1914	$ 50
April 1, 1914	500	Nov. 1, 1914	100
July 1, 1914	400		
Oct. 1, 1914	500		

Note. Take 1% for each of the four periods.

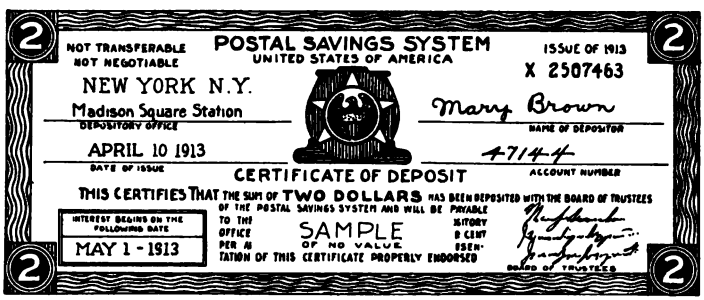
8. Find the balance due on the following, Jan. 1, 1915, with interest at 4 %, due the first of each month.

Deposits		Withdrawals	
Jan. 1, 1914	$200	Feb. 2, 1914	$100
Apr. 1, 1914	100	Aug. 4, 1914	100
June 1, 1914	100	Nov. 6, 1914	100
July 1, 1914	100		
Sept. 1, 1914	100		
Oct. 1, 1914	100		
Dec. 1, 1914	100		

POSTAL SAVINGS BANKS

The **postal savings** system was established by the United States government to provide opportunity for depositing small savings, with the security of the government for repayment.

Accounts may be opened with $1 or more by anybody 10 years of age or over. Smaller amounts than $1 may be saved for deposit by the purchase of postal savings cards and stamps.



Deposits are evidenced by **postal savings certificates** issued in denominations of $1, $2, $5, $10, $20, and $50, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins. Not more than $100 may be deposited in one month nor more than $500 all together, exclusive of interest.

Interest is allowed on all deposits at the rate of 2 per cent per annum, computed on each savings certificate separately, and payable annually. No interest is paid on money that remains on deposit for a fraction of a year only. Deposits bear interest from the *first day* of the month next following that in which they are made.

Compound interest is not allowed; but a depositor may withdraw interest payable and include it in a new deposit, which will bear interest at the regular rate.

A depositor is permitted to exchange the whole or any part of his deposits in sums of $20, $40, $60, $80, $100, or multiples of $100 up to and including $500, into United States registered or coupon bonds bearing interest at the rate of $2\frac{1}{2}$ per cent per annum, payable semiannually, and redeemable at the pleasure of the United States after one year from date of issue, both principal and interest payable 20 years from such date in United States gold coin. Such exchange may be made under date of Jan. 1 and July 1 of each year, provided such bonds are then available.

1. If you deposit $20 in a postal savings bank on Jan. 30, 1914, when does interest begin thereon and when will the interest payment be due? How much interest will be due?

2. When does interest begin on a deposit opened Jan. 15, 1914? Feb. 28, 1914? March 4, 1914?

3. If you deposit, on March 31, 1914, $85 in a postal savings bank, receiving certificates for $5, $10, $20, and $50, how much interest will be due you on April 1, 1915?

4. If a boy deposits $15 in a postal savings bank April 30, 1914, and $50 May 30, 1914, how much interest will be due him May 1, 1915 on the $15 deposit and June 1, 1915 on the $50 deposit?

5. If $500 worth of postal certificates are exchanged on Jan. 1, 1914 for United States bonds, how much interest will be due on them July 1, 1914 at $2\frac{1}{2}$% per annum?

6. If $350 worth of postal certificates are exchanged on July 1, 1914 for United States bonds, how much interest will be due on them Jan. 1, 1915 at $2\frac{1}{2}$% per annum?

EXCHANGE

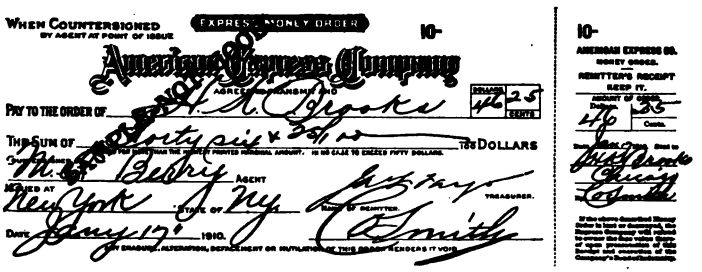
Exchange is the method of paying debts or collecting credits in distant places without transmission of money.

Exchange between two places in the same country is called **domestic exchange.**

Payments in the same country may be made by *postal money order*, by *express money order*, by *telegraphic money order*, by *bank check*, by *bank draft*, or by a *commercial draft* of a creditor on a debtor.

A **postal money order** is a written order by the postmaster in one place, to the postmaster in another place, to pay a specified sum of money to the person named in the order.

An **express money order** issued by an agent of an express company to another agent is similar to a postal money order. Both are negotiable.



Postal and express money orders, in addition to the sum specified on the face, cost as follows :

Orders for sums not exceeding	$2.50	3¢
Over $2.50 and not exceeding	5.00	5¢
Over 5.00 and not exceeding	10.00	8¢
Over 10.00 and not exceeding	20.00	10¢
Over 20.00 and not exceeding	30.00	12¢
Over 30.00 and not exceeding	40.00	15¢
Over 40.00 and not exceeding	50.00	18¢
Over 50.00 and not exceeding	60.00	20¢
Over 60.00 and not exceeding	75.00	25¢
Over 75.00 and not exceeding	100.00	30¢

NOTE. Postal orders are not issued for sums larger than $100.

A **telegraphic money order** is similar to the two orders mentioned. In addition to the regular charge for a 15-word message between the two places, the rates are:

For orders for not more than $25	25 cents
For more than $25 but not more than $50	35 cents
For more than $50 but not more than $75	60 cents
For more than $75 but not more than $100	85 cents

After the first $100 up to and including $3000, add 25 cents for each $100 or part thereof.

A **bank check** is an order on a bank, given by one of its depositors, to pay a specified sum to the order of a certain person. (See p. 357.)

A **certified check** is a notice to the payee that the amount of the check has been taken from the maker's deposit and placed with the bank's funds for the payment of the check when presented. Certified checks are used when the payee does not wish to accept a personal check.

As a rule, a small fee, called **exchange**, is charged for collecting a check in any town outside the location of the bank on which the check is drawn.

A **bank draft**, or **bill of exchange**, is a check drawn by one bank upon another.

By whom is the draft on p. 371 drawn? For what sum is it drawn? on what bank?

BANK DRAFT

No. 769

Third National Bank

Fort Wayne, Ind., June 8, 1914.

Pay to the order of Robert Saunders $700.00

Seven hundred $\frac{no}{100}$ Dollars.

To Corn Exchange Bank

NEW YORK, N. Y.

Frank Thompson,

Cashier.

Banks usually keep some money on deposit in other banks, called **correspondence banks**, located in large cities. Against this money they draw drafts, which they sell to customers who desire to remit payments to such cities. A small charge, called **exchange**, is usually made by banks for such service. This charge ranges from 10 ¢ to 50 ¢ on small drafts, and from $\frac{1}{10}$ % to $\frac{1}{4}$ % on drafts for larger amounts.

SIGHT COMMERCIAL DRAFT

3000\frac{no}{100}$ Milwaukee, Wis., April 10, 1914.

At sight pay to the order of Traders' Bank of Milwaukee

Three thousand $\frac{no}{100}$ Dollars.

Value received, and charge to account of

To Henry Smith, William Brown.

533 Wabash Ave., Chicago, Ill.

Collections may also be made by **commercial draft.** If Henry Smith of Chicago owes William Brown of Milwaukee $3000, which Mr. Brown wishes to collect at once, he may draw on Henry Smith a sight draft like the one on p. 371.

The Milwaukee bank sends this draft to some Chicago bank, which collects the amount of Henry Smith and remits it to the Milwaukee bank, which in turn pays the amount, less a small fee for collection, to William Brown.

The following draft, due "Ninety days after sight," is called a **time draft.**

TIME COMMERCIAL DRAFT

3000\frac{00}{000}$ Milwaukee, Wis., April 10, 1914.

Ninety days after sight Pay to the order of Traders' Bank of Milwaukee

Three thousand $\frac{no}{100}$ Dollars.

Value received, and charge to account of

To Henry Smith, William Brown.

533 Wabash Ave., Chicago, Ill.

If Mr. Smith intends to pay the draft when presented to him, he writes across the face in red ink "*Accepted*," and then adds the date and his signature. This draft has now the force of a promissory note and may be discounted, for the time it has to run, after date of acceptance.

Formerly, if trade between two trade centers like Chicago and New Orleans was equal, exchange was at par. If Chicago owed New Orleans, the demand in Chicago for drafts on New Orleans was greater than the demand in New Orleans for Chicago drafts. Hence, the drafts were at a premium in Chicago. Under reversed conditions, they were at a discount. For a long time, however, there has been little variation of this kind.

PROBLEMS

WRITTEN

1. Find the cost of a postal or an express money order for \$2.35; \$2.55; \$8; \$55.75; \$98.60.

2. Find the cost of sending a telegraphic money order for \$2345, allowing \$.75 for the message.

3. What must be paid in Indianapolis for a draft on Chicago for \$800, at $\frac{1}{10}$ % exchange?

SOLUTION. $\frac{1}{10}$ % of \$800 = \$.80, cost of exchange.
\$800 + \$.80 = \$800.80, cost of draft for \$800.

4. A merchant in Chicago owes a merchant in Fort Wayne \$4750. How much must be paid in Chicago for a draft to settle the account, at $\frac{1}{4}$ % exchange?

5. How much must be paid for a draft of \$354 at $\frac{1}{8}$ % exchange?

6. Find the proceeds of a draft for \$5000, payable 30 days after sight, if discounted at 6 % for the full term.

SOLUTION. The bank discount on \$1 for 30 da. at 6 % = \$.005
5000 × \$.005 = \$25, bank discount on \$5000.
\$5000 − \$25 = \$4975, proceeds.

7. Find the proceeds of a draft for \$21,000, for ninety days after date, if discounted at 5 % for the full term.

8. T. C. Arnold & Co. of New York drew a sight draft for \$2000 on R. J. Steele of Evansville, Ind. This draft was sold at a bank at $\frac{1}{4}$ % discount. Find the proceeds.

9. Find the proceeds of a sight draft for \$3000, exchange and collection together being $\frac{1}{8}$ %.

10. Find the proceeds of a draft for \$12,000, for sixty days after date, if discounted at 6 % for the full term.

11. Mr. Bruce sends Mr. Jameson a check for $480 in payment of a debt. Mr. Jameson cashes the check at a bank which charges him $\frac{1}{10}\%$, for collection. How much does the bank pay Mr. Jameson?

New York, Chicago, and San Francisco are the principal exchange or money centers in the United States.

12. Find the cost in Chicago of a draft on New York for $2500 if $1.00 per $1000 is charged for collection.

13. Find the cost in South Bend, Indiana, of a draft on New York for $5000 if $\frac{1}{10}\%$ is charged for collection.

14. Find the cost in Chicago of a draft for $3500 on Vincennes, Indiana, if $1.80 per $1000 is charged for collection.

People intending to travel abroad often secure a letter of credit or a foreign express check.

A **letter of credit** is a letter from a bank in one country to one or more of their correspondent banks in another country, directing them to pay to the holder any sum not exceeding a certain amount specified in the letter.

Travelers' checks are guaranteed checks issued by express companies, banks, steamship companies, etc., in denominations of $10, $20, $50, $100, and $200, at a fixed rate of $\frac{1}{2}\%$ of the face value.

The following values were printed on a $20 check:

£	s.	d.	Francs	Marks	Lire	Crowns	Florins
4	1	2	102.50	82.50	102.50	73.39	49.02

15. Find how much English, French, and German money would be received in return for a $50 check; for a $10 check; for a $100 check.

Note. There are 12 **pence** (*d.*) in one **shilling** (*s.*) and 20 **shillings** (*s.*) in 1 **pound** (£).

STOCKS AND BONDS

A **company** is an association of individuals for the transaction of business.

A **corporation** is a company governed in its operations by a general law or a special charter.

A **charter** is the legal act of incorporation. It states the powers and duties of the corporation.

The **stock** of a company is the money invested by it to carry on its business. The owners of stocks are called **stockholders.**

A **share** is one of the equal parts into which the stock has been divided. A share is usually of the value of $100; but may be more or less. In this book shares are $100 in value.

The **face,** or **par value,** is the sum for which the share is issued.

The **market value** is the sum for which a share can be sold.

A **dividend** is a sum paid to the stockholders out of the gains of the company. It is computed as a per cent of the face or par value of the stock.

An **assessment** is a sum required of the stockholders, according to the amount of stock held by each, to meet losses or expenses of the company.

Corporations often issue two kinds of stock, called **preferred stock** and **common stock.** The holders of preferred stock generally receive a specified rate of dividend that is paid before the holders of common stock receive any share of the profits.

Bonds are promissory notes issued by corporations and secured by mortgages on some property owned by the corporation. The usual face value of a bond is $1000.

Coupon bonds have interest certificates attached to them, which are redeemed as the interest comes due. They are payable to bearer. When interest is due the coupon is cut off and may be cashed at a bank or at the place where issued.

Registered bonds are recorded in the name of the owner in the books of the company issuing them. They cannot be transferred without indicating the transfer in the company's records.

Par and market value in bonds have the same meaning as in stocks.

NOTE. **Stockholders** are owners of the property, while **bondholders** are creditors.

Stocks and bonds are **at par** when they sell for the face value; **above par**, or at a premium, when they sell for more than the face value; **below par**, or at a discount, when they sell for less than the face value.

STOCK CERTIFICATE



Bonds bear interest at the rate specified on the face. It is always computed on the par value. A 4% bond is one that bears $40 interest per $1000 bond in one year. U. S. 5-20's are United States bonds bearing 5% interest, and running for 20 years.

A **broker** is a person who buys and sells stocks and bonds for another.

Brokerage is the compensation of the broker. It is computed on the par value. "Brokerage $\frac{1}{8}$" means that the broker receives $\frac{1}{8}$ of a dollar for each $100 share bought or sold; that is, $\frac{1}{8}$% of the par value.

1. What is the name of the corporation that issued the stock certificate represented on p. 376? Where is this corporation located? What is the amount of the capital stock? What is the par value of each share? Who is the owner of this certificate? How many shares of the stock are represented by it? What is its face value?

2. If the company declares a 5% dividend, what is Mr. Fluker's income from the stock?

3. If he sells it at a premium of 10%, how much will he receive for it?

QUOTATIONS

STOCKS		BONDS	
Baltimore and Ohio	92¾	U. S. 4s coup., 1925	112
Baltimore and Ohio pfd.	98	U. S. 3s reg., 1918	105
Pennsylvania	107	Can. So. 5s	104

The above quotations from a newspaper mean: Baltimore and Ohio common stock, at 92¾% of its par value; Baltimore and Ohio preferred stock, at 98% of its par value; Pennsylvania railroad common stock at 107%; United States coupon bonds, paying 4% interest, and due in 1925, at 112%; United States registered bonds, paying 3% interest and due in 1918, at 105%; and Canada Southern railroad bonds, paying 5% interest, at 104%.

Study Recitation

1–24. Find the cost of the following stocks and bonds, including $\frac{1}{8}\%$ for brokerage.

Note. The par value of 1 share of each stock is $100 and the par value of each bond is $1000.

Stocks				Bonds			
	No. of Shares	Name of Stock	Market Price		No. of Shares	Name of Bond	Market Price
1.	10	Amalgamated Copper	$70\frac{7}{8}$	*13.*	4	U. S. 3s coup.	$102\frac{3}{8}$
2.	5	Atchison, Topeka & Santa Fé	92	*14.*	5	Adams Ex. 4s	$73\frac{7}{8}$
				15.	10	Jap. 1st $4\frac{1}{2}$s	$88\frac{1}{4}$
3.	10	Baltimore and Ohio	$92\frac{3}{4}$	*16.*	3	N. Y. Canal 4s	$98\frac{1}{2}$
4.	5	Canadian Pacific	223	*17.*	2	N. Y. City 4s	$96\frac{3}{8}$
5.	10	Central Leather	24	*18.*	6	Balt. & O. 4s	$91\frac{1}{8}$
6.	100	Consolidated Gas	130	*19.*	10	Can. So. 5s	104
7.	20	Erie	27	*20.*	8	N. Y. Cen. $3\frac{1}{2}$s	$80\frac{1}{8}$
8.	30	Illinois Central	106	*21.*	5	Southern Railway 5s	$102\frac{3}{4}$
9.	50	Lehigh Valley	$148\frac{1}{4}$				
10.	100	Pennsylvania	107	*22.*	4	Union Pacific 4s	$85\frac{7}{8}$
11.	10	Southern Pacific	$85\frac{7}{8}$	*23.*	10	Wabash 1st 5s	$101\frac{1}{2}$
12.	5	United States Steel	56	*24.*	3	West Shore 4s	$90\frac{1}{2}$

Solution to Example 1

Cost of 1 share at $70\frac{7}{8} + \frac{1}{8} = .71$ of $100, or $71.
Cost of 10 shares = 10 × $71, or $710.

Solution to Example 13

Cost of 1 bond at $102\frac{3}{8} + \frac{1}{8} = 102\frac{1}{2}\%$ of $1000, or $1025.
Cost of 4 bonds = 4 × $1025, or $4100.

25–48. Find the proceeds from the sale of the above stocks and bonds, deducting $\frac{1}{8}\%$ for brokerage.

Solution to Stock in 1

Proceeds from sale of 1 share at $70\frac{7}{8} - \frac{1}{8} = .70\frac{3}{4}$ of $100, or $70.75.
Proceeds from sale of 10 shares = 10 × $70.75, or $707.50.

SOLUTION to Bond in 13

Proceeds from sale of 1 bond at $102\frac{1}{8} - \frac{1}{8} = 1.02\frac{1}{4}$ of $1000, or $1022.50.
Proceeds from sale of 4 bonds = 4 × $1022.50, or $4090.

49–60. Find the yearly interest on bonds numbered 13 to 24.

SOLUTION to Bond in 13

3% of $1000 = $30; 4 × $30 = $120.

61. If I buy a $1000 5% bond at 102, which includes brokerage, what rate of income, to the nearest tenth of 1 per cent, do I realize on my investment?

SOLUTION

The interest on the bond is $50 a year. The investment is $1020. $50 is $\frac{50}{1020}$, or 4.9%, of $1000. *Ans.* 4.9%.

WRITTEN

62. Add $\frac{1}{8}$% brokerage to each of the quotations on bonds in numbers 13 to 24, and find the rate of income on each to the nearest tenth of 1 per cent.

63. How much must be paid, including $\frac{1}{8}$% brokerage, for 15 shares of National Lead Co. stock at $106\frac{7}{8}$?

64. If I buy 10 shares of Northern Pacific Stock at $116\frac{3}{8}$, with $\frac{1}{8}$% brokerage, and sell it at $120\frac{5}{8}$, with $\frac{1}{8}$% brokerage, how much do I gain?

65. How much yearly interest will be received on five U. S. 3's and what rate of income will they yield when bought at 102?

66. How much must I pay for a 4% bond to realize 5% on my investment?

SOLUTION. $40, the yearly interest on the bond, is 5% of the cost of the bond, $40 ÷ .05 = $800; hence the bond would be quoted at 80.

67. Make and solve problems like the above, using quotations from a daily newspaper.

GENERAL REVIEW

PROBLEMS

Oral

1. John had 36 pencils and used $33\frac{1}{3}\%$ of them. How many had he left?

2. A knife costing \$.80 is sold at a profit of 25%. What is the selling price?

3. What per cent of 30¢ is 30¢?

4. I buy a watch for \$16 and sell it for \$24. Find the per cent of gain.

5. A merchant, after deducting 10% from the list price of an article, receives \$2.70 for it. Find the list price.

6. James sells an article for \$.96, thereby gaining $33\frac{1}{3}\%$ of the cost. What is the cost?

7. By selling apples at 4¢ a dozen more than cost, a boy gained 10%. How much did he pay per dozen?

8. How much is paid for a carriage listed at \$240, with discounts of 25% and 10%?

9. By selling 3 qt. of milk for the price of 1 gal., I gain what per cent?

10. If Mr. Shaw buys lemons at 3 for 2¢ and sells them at 3 for 5¢, what per cent does he gain?

11. Mr. Baker has a 5% mortgage of \$10,000 on Mr. Beard's house. What amount of interest must Mr. Beard pay him each 6 months?

12. If a gain of \$500 on some goods is a gain of 20%, what is the cost of the goods?

13. A farmer set out 30,000 celery plants, but only $83\frac{1}{3}\%$ of them matured. If these were sold at \$3 per hundred, how much did the farmer receive for them?

14. Mr. Bruce buys 10 shares of stock at 80 and sells them at 100. Find the cost and the selling price, including $\frac{1}{8}\%$ for brokerage in each case.

15. Find the commission at 5% on a sale of $850.

16. $40 is 200% of what sum?

17. An agent buys a lot for $1500 and charges 2% commission. Find the entire cost of the lot.

18. A building worth $4000 is insured for $\frac{7}{8}$ of its value at 80¢ a hundred. Find the premium.

19. A hardware merchant sold two knives for 90¢ each. On one he lost 25%, and on the other he gained 25%. Did he lose or gain on the transaction, and how much?

20. An insurance company charged $240 premium for insuring a store for $\frac{3}{4}$ of its value at $1 per hundred. Find the value of the store.

21. A school district having an assessed valuation of $50,000 wishes to raise a tax of $250. What is the tax rate?

22. What is the tax of A, living in the school district mentioned, if his property is assessed at $5000?

23. What is the duty on 1000 yards of Brussels carpet, invoiced at $2.40 a yard, at 25% ad valorem?

24. What is the interest on $500, at 5%, for three years?

25. Find the amount due on a $600 note, bearing 6% interest, for 2 years and 6 months.

26. Find the bank discount at 8% on a $1000 note, due in 3 months, without interest.

27. Find the cost of a $1000 bond quoted at $98\frac{7}{8}$ with $\frac{1}{8}\%$ brokerage.

28. Find the yearly interest on a $1000 5% bond.

29. Find the cost of a draft for $1000 at $\frac{1}{10}\%$ exchange.

WRITTEN

30. Mr. Brown sold his farm for $6600, thereby gaining 20%. What was his gain?

31. A horse costing $125 is sold for $140. Find the gain per cent.

32. 25% of the cost is lost in selling an article for $4.50. Find the loss.

33. Find the commission at 2% for selling 6 carloads of hogs, weighing in all 90,000 pounds, at 8 ¢ a pound.

34. An agent receives $160 as a 2% commission for the purchase of a lot. Find the price paid for it.

35. A commission merchant purchases $15,500 worth of cotton, and charges 2½% commission. What is the amount of the bill he sends to his employer?

36. Find the number of days between Jan. 4, 1914 and August 10 of the same year.

37. Find the exact interest on $450 at 4½% from Jan. 4, 1914, to Aug. 10, 1914.

38. Find the bank discount on a note amounting to $8020, discounted for 65 days, at 8%.

39. Mr. Smith sold his farm for $6360, thereby gaining 20%. Would he have gained or lost, and what per cent, if he had sold it at $5500?

40. If 5% is lost by selling an article for $2.47, what per cent is gained or lost by selling it for $2.99?

41. A stock of goods worth $16,400 is insured for ¾ of its value at 8½%. What is the premium? If the entire stock should burn, what would be the owner's loss by fire?

42. Find the cost of 10 shares of stock quoted at 95½, brokerage ⅛%.

43. Find the semiannual interest on 5 U. S. 3's, par value $1000.

44. A merchant buys through an agent 7300 yd. of carpeting at $1.25 a yard, paying $\frac{3}{4}\%$ commission. The freight amounts to $7.37. At what price per yard must the carpet be sold to realize a profit of 20%?

45. An agent sold 13,125 bu. of wheat at $.80 a bushel, on 5% commission. What was his commission?

46. If 24 bu. of corn are raised from 1 pk. of corn, what per cent is the increase? What is the increase?

47. A farmer buys 24 head of cattle at $80 a head, and after losing 6 sells the remainder at $105 a head. What per cent does he gain or lose?

48. A grocer selling eggs at 20 cents a dozen cleared 25% of the cost. How much did the eggs cost him? Another grocer, selling at the same price, cleared $33\frac{1}{3}\%$ of the cost. What was the cost?

49. A merchant bought silk at $1.50 a yard. He marked it so as to give him a profit of 20%. As it did not prove a good seller, he offered it at a discount of $33\frac{1}{3}\%$ and 10%. What was his asking price? his selling price? his loss per cent?

50. A farmer bought 40 acres of land at $40 an acre and spent $600 for improvements. For how much must he sell it to gain 20%?

51. I receive $4000 as commission at 4% on a sale of goods. Find the amount of sale.

52. A merchant sold a house for $3680, losing 8%. For how much should he have sold it to gain $12\frac{1}{2}\%$?

53. If a lamp bought for $\frac{1}{8}$ less than its value is sold for $\frac{1}{4}$ more than its value, what is the per cent of gain?

54. A hotel is insured for $\frac{3}{5}$ of its value, at $1\frac{1}{2}\%$ premium, and the premium amounts to $200. What is the value of the hotel?

55. I sold two machines for $300 each. On one I gained 20 % and on the other I lost 20 %. Did I gain or lose on the transactions, and how much?

56. A merchant bought a bankrupt's stock at 45 cents on the dollar, and sold it at 10 % below the original price. What per cent did he gain?

57. An article lost 10 % by wastage, and was sold for 30 % above cost. What was the gain per cent?

58. What per cent of 100 bushels are 50 pecks?

59. A dealer bought an article at 10 % below the price asked, and sold it for $2400, gaining $33\frac{1}{3}\%$. What was the price asked?

60. A commission merchant receives 17 firkins of butter, each containing 43 lb., 630 lb. of chickens, and 68 cases of eggs, of 16 doz. each. He sells the butter at 30 cents a pound, the chickens at 24 cents a pound, and the eggs at 30 cents a dozen. Find the amount of his commission at 4 %.

61. If 3 % more is gained by selling a horse for $83.25 than by selling it for $81, what is the cost of the horse?

62. If a boy gains 12 % by selling 5 oranges for 14 cents, what per cent would he gain by selling them at 6 for 18 cents?

63. Mr. Jones insured his house, worth $4800, for 3 years for $\frac{5}{6}$ of its value, at 75 cents per hundred. Find the rate per cent charged, the premium, and the company's loss, if the house should burn.

64. Find the duty, at 40 % ad valorem, on an invoice of 800 doz. linen handkerchiefs, valued in Berlin at $3.75 a dozen.

65. A tax of $25,000 is assessed in a city upon a property valuation of $125,000,000. What is the rate of taxation, and what does Mr. T. Gray pay, whose store is assessed at $16,500, house at $2250, and personal property at $950?

66. What price must be asked for goods that cost $240 in order that there may be a clear profit of $16\frac{2}{3}$ % after allowing a discount of $12\frac{1}{2}$ % from the list price?

67. A 4 % note of $400 was given March 8, and paid December 15 of the same year. Find the amount of the note.

68. Write a negotiable note for $300, bearing interest at the rate of 7 % for 2 years, drawn by G. J. Jones in favor of R. S. Brown, July 5, 1913. Underscore the words that make it negotiable. What must R. S. Brown do if he sells the note to M. A. Starr?

69. Find the proceeds of the note in problem 68 if it is discounted at a bank Jan. 3, 1914 at 8 %.

70. What should the Mechanics' Bank pay for a note of $1500, dated July 12, due in 4 months, bearing 8 % interest, discounted Sept. 1 at 6 %?

71. A merchant is given the choice of paying $500 in 6 months, or receiving a 2 % discount for cash. Which is the better offer, and by how much, if money is worth 6 %?

72. How much interest will be paid annually on a $1000, 5 % bond? a $4\frac{1}{2}$ % bond? a 4 % bond?

73. What rate of income do I realize on a $1000 4 % bond bought at 80?

74. Find the cost of 5 $100 shares of stock bought at $68\frac{1}{8}$ with brokerage at $\frac{1}{8}$ %.

Problems without Numbers

75. If you know the principal, the rate, and the time, how can you find the interest and the amount?

76. If you know the principal, the rate, and the interest, how can you find the time?

77. If you know the principal, the interest, and the time, how can you find the rate?

78. If you know the interest, the rate, and the time, how can you find the principal?

79. Write out and indorse a check made payable to Robert Hall by Frank Clark.

80. Write a negotiable promissory note.

81. If you know the amount of a mortgage and the rate of interest, how can you find the semiannual interest due on the mortgage?

82. If you know the face of a note, the term of discount, and the rate, how can you find the bank discount? the proceeds?

83. How can you find how much interest is due each half year on a savings bank account?

84. When does interest begin on a deposit made in a postal savings bank?

85. How can you pay money at a distance?

86. If you know the face of a draft and the rate of exchange, how can you find the cost of the draft?

87. If you know the market price of a stock and the rate of brokerage, how can you find the cost of a given number of shares? the proceeds from the sale of a given number of shares?

88. If you know the denomination of a bond and the rate of interest, how can you find the yearly interest?

89. If you know the market price of a bond and the yearly interest, how can you find the rate of income it yields?

USE OF SYMBOLS

In the analysis of problems it is frequently necessary to perform long operations in addition, subtraction, multiplication, or division in order to find the several results that must be obtained to secure the final result.

The analysis of the following problems illustrates the necessity of such operations in order to find the answer.

1. If 125 acres of land are bought for $62.50 an acre and sold for $9375, what is the gain per acre?

ANALYSIS. If land costs $62.50 an acre, 125 acres cost 125 × $62.50, or $7812.50. If land costing $7812.50 is sold for $9375, the gain equals $9375 − $7812.50, or $1562.50. If $1562.50 is gained by the sale of 125 acres of land, the gain on 1 acre is $1562.50 ÷ 125, or $12.50.

In this analysis three operations are required and three results are found. $62.50 must be multiplied by 125; $7812.50 must be subtracted from $9375; $1562.50 must be divided by 125.

When pupils have been trained to add, subtract, multiply, and divide accurately and rapidly, it is entirely unnecessary to have *all* the problems in the arithmetic worked out completely, as the analysis demands all the reasoning involved in the solution. Much time usually given to figuring in arithmetic can thus be saved for work in other subjects.

In the solutions of problems, any letter may be substituted for, or used instead of, any number which might be the result of any operation, provided the same letter is not used instead of any other number in the solution of the problem. Numbers resulting from other operations in the same solution may be represented by other letters.

The use of letters for numerical results of indicated operations is illustrated in the following analysis of problem 1.

ANALYSIS. If land costs \$62.50 an acre, 125 acres cost 125 times \$62.50, or x dollars. If land costing x dollars is sold, at a gain, for \$9375 the gain equals \$9375 $-$ x dollars, or y dollars. If y dollars are gained by the sale of 125 acres of land, the gain on one acre is $\frac{1}{125}$ of y dollars, or y dollars $\div$ 125, or $\frac{y}{125}$ dollars. The expression $\frac{y}{125}$ dollars represents the answer. If desired, z dollars may be substituted for $\frac{y}{125}$ dollars.

In an analysis of this kind, wherever we indicate an operation, we at once assume it to have been performed and substitute a letter for the result. As we proceed with the analysis, whenever it is necessary to use this result, we use instead the substituted letter, and substitute another letter for the result of the next operation, and so on until the expression or letter representing the final result is found. We can thus proceed with the analysis without the delay or interruption of our course of reasoning, resulting from the performance of each operation.

This analysis might also be indicated as follows:

$$\frac{\$9375 - (125 \times \$62.50)}{125} = x \text{ dollars gain. } \textit{Ans.}$$

A number of illustrations of the first form of analysis are here given. The same method may be used at the teacher's discretion with other problems throughout the book.

2. What change should be received from \$50, in payment for 16 doz. buttons at \$.36 a dozen, 6 yards of velvet at \$4.75 a yard, and 15 yards of lining at \$.18 a yard?

ANALYSIS. 16 doz. buttons at \$.36 a dozen cost 16 times \$.36, or x dollars. 6 yards of velvet at \$4.75 a yard cost 6 times \$4.75, or y dollars. 15 yards of lining at \$.18 a yard cost 15 times \$.18, or z dollars. The cost of the buttons, velvet, and lining equals $x + y + z$, or p, dollars. The amount of change to be received is \$50 $-$ p dollars or r dollars.

Or, \$50 $-$ (16 $\times$ \$.36 + 6 $\times$ \$4.75 + 15 $\times$ \$.18) = x dollars. *Ans.*

3. At \$1.50 a day, how much does a man earn in 3 weeks, if he works $5\frac{3}{4}$ days the first week, $4\frac{3}{5}$ days the second week, and $5\frac{2}{3}$ days the third week?

ANALYSIS. During the 3 weeks he works $5\frac{1}{4}$ days + $4\frac{1}{8}$ days + $5\frac{2}{3}$ days, or x days. If he earns \$1.50 a day, in x days he earns x times \$1.50, or y dollars.

Indicate this analysis in another way.

4. A drover with \$2144 bought as many horses as possible for \$165 each, and spent the remainder for sheep at \$4 a head. How many of each did he buy?

ANALYSIS. If horses cost \$165 each, for \$2144 a drover could buy $\frac{\$2144}{\$165}$, or x horses, and have y dollars left. At \$4 each, for y dollars he could buy $\frac{y}{4}$, or z sheep. He bought x horses and z sheep.

5. Corn weighs 56 lb., and wheat 60 lb., a bushel. If a farmer sells 6640 lb. corn at \$.35 a bushel, and 4290 lb. wheat at \$.60 a bushel, how much should he receive?

ANALYSIS. 6640 lb. corn = $118\frac{4}{7}$ bu., or x bu. x bu. corn at \$.35 a bushel are worth $x \times \$.35$, or y dollars. 4290 lb. wheat = $71\frac{1}{2}$ bu., or z bu. z bu. wheat at \$.60 a bushel are worth $z \times \$.60$, or r dollars. The farmer should receive y dollars + r dollars, or s dollars.

Or, $118\frac{4}{7} \times \$.35 + 71\frac{1}{2} \times \$.60 = x$ dollars. *Ans.*

6. A bin 8 ft. square and 6 ft. deep is $\frac{2}{3}$ full of wheat. What is the value of the wheat at \$.70 a bushel?

ANALYSIS. If the bin is 6 ft. in depth and is $\frac{2}{3}$ full, the depth of the wheat in the bin is $\frac{2}{3}$ of 6 ft., or 4 ft. If the mass of wheat in the bin is 8 ft. wide, 8 ft. long, and 4 ft. deep, its volume is $8 \times 8 \times 4$ cu. ft., or 256 cu. ft. 256 cu. ft. = 256×1728 cu. in., or x cu. in. One bushel contains 2150.4 cu. in. x cu. in. = $\frac{x}{2150.4}$ bushels, or y bushels. The value of y bushels of wheat at \$.70 a bushel = $y \times \$.70$, or z dollars.

As in the first part of the above analysis, whenever the results of the indicated operations can be found mentally without trouble, letters should not be substituted for such results.

7. The interest of any sum is equal to the product of the principal by the rate by the time expressed in years. Ex-

press this rule by a formula using p for principal, r for rate, and t for time.

8. A lot 120 ft. long and 80 ft. wide is surrounded by a sidewalk 8 ft. wide. What will be the cost of the sidewalk at $1.50 per square yard?

ANALYSIS. Since the sidewalk is 8 ft. wide, the length of each end walk, including the corners, is 80 ft. + 8 ft. + 8 ft., or 96 ft. The length of both end walks is 2 × 96 ft., or 192 ft. The length of each side walk is 120 ft.: of both side walks is 2 × 120 ft., or 240 ft. The entire length of the walk is 192 ft. + 240 ft., or x ft. The area of the walk is $x \times 8$, or y sq. ft. As there are 9 sq. ft. in a square yard, the number of square yards in the walk is $\frac{y}{9}$. At $1.50 per square yard, the cost of $\frac{y}{9}$ sq. yd. is $\frac{y}{9} \times \$1.50$, or z dollars.

The above problem may be analyzed correctly in a number of ways, in any one of which letters may be used for the results that cannot be easily found mentally.

9. How much will a planter receive who sends to market 127 bales of cotton, averaging 500 pounds, if the cotton is sold at $\$.09\frac{3}{4}$ a pound, the freight being $.57 a hundredweight, drayage $60, and commission $3\frac{1}{2}$ %?

ANALYSIS. If the bales average 500 pounds, 127 bales weigh 127 × 500 pounds, or x pounds. x pounds, at $\$.09\frac{3}{4}$ per pound, sell for $x \times \$.09\frac{3}{4}$, or y dollars. The commission for selling is $.03\frac{1}{2}$ of y dollars, or z dollars. As the freight rate is $.57 a hundred pounds, the expense for freight on x pounds is $\frac{x}{100} \times \$.57$, or r dollars. The entire expense of selling is, — commission, z dollars, + freight, r dollars, + drayage, $60, or z dollars + r dollars + $60, or s dollars. Since the cotton sells for y dollars and the expense for selling is s dollars, the planter will receive y dollars − s dollars, or t dollars.

10. What is the rule for finding the area of a rectangle? Express it in a formula, using A for area, b for base, and h for height or altitude.

EXPRESSING EQUATIONS

STUDY RECITATION

1. If x represents the number of dollars A earns in 1 day, 3 times x, which may be written $3x$, will represent the number of dollars he can earn in 3 days at the same rate. $\frac{x}{2}$ will represent the number of dollars he can earn in half a day, and $\frac{5}{2}x$ or $\frac{5x}{2}$ will represent the number of dollars he can earn in $2\frac{1}{2}$ days, or $\frac{5}{2}$ days. What, then, will represent the number of dollars he can earn in 2 days? in 5 days? in 10 days? in $\frac{1}{3}$ of a day? in $2\frac{1}{3}$ days? in $4\frac{1}{4}$ days?

2. How many are $1¢ + 5¢ + 6¢$? $1x + 5x + 6x$? ($1x$ or once x is written simply x.)

3. If John has x dollars and Henry has 4 times as many dollars, how many dollars has Henry? How many dollars have John and Henry? (*Ans.* Once x plus 4 times $x = 5$ times x, or $x + 4x = 5x$.) How many more dollars has Henry than John? (*Ans.* $4x - x = 3x$.) The number of dollars John has is what part of the number Henry has? $\left(\textit{Ans. } x \text{ is } \frac{1}{4} \text{ of } 4x, \text{ or } \frac{x}{4x} = \frac{1}{4}.\right)$

4. John has 2 more apples than James. If we let x equal the number James has, what is the expression for the number John has? (*Ans.* $x + 2$.) If we let x equal the number John has, how many has James? (*Ans.* $x - 2$.)

5. If Henry has $\frac{1}{2}$ as many apples as Fred has, and Fred's are represented by x, what is the expression for the number Henry has? $\left(\textit{Ans. } \frac{x}{2}.\right)$ If Fred gives Henry 2 apples, what is the expression for the number Fred then has? (*Ans.* $x - 2$.) For the number Henry then has? $\left(\textit{Ans. } \frac{x}{2} + 2.\right)$

6. If A has x dollars and B has \$8 more than A, how many dollars has B? (*Ans.* $x + 8$.) If C's money is \$8 less than A's, how many dollars has C? (*Ans.* $x - 8$.) If D's money is 8 times A's, how many dollars has D? (*Ans.* $8x$.) If E's money is $\frac{1}{8}$ of A's, how many dollars has E? $\left(\textit{Ans.}\ \frac{x}{8}.\right)$ If F's money is \$2 more than 8 times A's, how many dollars has F? (*Ans.* $8x + 2$.) If G's money is \$2 less than 8 times A's, how many dollars has G? (*Ans.* $8x - 2$.)

7. If James has 8 apples and William has 6 apples, they both have $(8 + 6)$ apples. If Henry has $\frac{1}{2}$ as many apples as both James and William, he has $\frac{8+6}{2} = \frac{8}{2} + \frac{6}{2} = \frac{14}{2}$, $= 7$ apples. Substitute x for the number of apples James has, and y for the number William has, and find the expression for the number Henry has.

8. What number added to 8 will equal 10? This may be stated $? + 8 = 10$. The **known numbers** here are 8 and 10, and the **unknown number** represented by ? is the answer 2.

If we represent the unknown number by x, the statement is $x + 8 = 10$. This is an **equation**, since what precedes the equality sign equals what follows it. The part on the left, $x + 8$, is called the **first member**, and the part on the right, 10, the **second member.**

WRITTEN

Make equations expressing the following, representing the unknown number by x:

9. If \$4 is added to Frank's money, he will have \$6.

10. James, who has \$3 less than John, has \$6.

11. Five times Ida's age equals Eva's age, 25 years.

12. One fourth of Anna's age equals 4 years.

13. What number increased by 7 equals 25?

14. The sum of two numbers is 30, and one of the numbers is 10.

15. The difference of two numbers is 5, and the larger number is 17.

16. Twice a number increased by 4 equals 50.

17. One fourth a number diminished by 3 equals 10.

18. One half of a number plus one fourth of it equals 8.

19. Two pieces of lead weigh 35 pounds, and one piece weighs 4 pounds more than the other.

20. If a piece of work can be done in 6 days, what part of it can be done in 1 day? in 3 days? in y days? $\left(Ans.\ \frac{1}{6};\ \frac{3}{6};\ \frac{y}{6}.\right)$ If a piece of work can be done in x days, what part of it can be done in 1 day? in 3 days? in y days? $\left(Ans.\ \frac{1}{x};\ \frac{3}{x};\ \frac{y}{x}.\right)$

21. If A can do a piece of work in 4 days, and B can do the same piece of work in 6 days, what part can each do in 1 day? What part can both do in 1 day? If x equals the number of days it takes A and B to do the work together, what is the expression for the part they can both do in 1 day? From these expressions form an equation.

22. If 6 bushels of potatoes cost \$3, the expression for the part of a dollar 1 bushel costs is $\frac{3}{6}$. The expression for the number of bushels \$1 will buy is $\frac{6}{3}$. If c bushels of potatoes cost d dollars, what is the cost of 1 bushel? How many bushels will \$1 buy? How many bushels will \$2 buy?

23. The sum of three numbers is 21. The second is twice the first and the third is twice the second. What is the ex-

pression for the second? for the third? for the sum of the first and the second? for the sum of the three? for the difference between the first and the second?

24. The sum of two numbers is 25. If the first number is 8, what is the expression for the second? If the first number is b, what is the second?

25. The sum of two numbers is a. The first number is 6. What is the second?

26. A father's age is three times that of his son. Make an expression for the age of each. What is the expression for the age of each 10 years later? The son is then one half as old as his father. Using the son's age, make an expression for the father's age. What two expressions are equal? Put them in the form of an equation. State the problem which the solution of the equation will answer.

SOLVING EQUATIONS

STUDY RECITATION

To **solve an equation** is to find the value of the unknown number.

1. Solve the equation $x + 8 = 10$.

In solving equations it is best, by transposing, to get the unknown numbers represented by letters alone in the first member and the known numbers in the second member.

SOLUTION. Since 10 is 8 more than x, x must be 8 less than 10; $x = 10 - 8$, or 2. PROOF. $2 + 8 = 10$.

2. Solve the equation $x - 8 = 10$.

SOLUTION. Since 8 less than x equals 10, x must equal 8 more than 10; $x = 10 + 8 = 18$. PROOF. $18 - 8 = 10$.

Notice in each of the preceding solutions that the known number is transposed from the left to the right member of the equation by changing the sign preceding it. In example 1, $+8$ is changed to -8 on being transposed. In example 2, -8 is changed to $+8$ on being transposed.

Any term may be transposed from one member to another of an equation, if the sign $+$ preceding the term is changed to $-$ or the sign $-$ to $+$.

Find the value of x in the following:

3. $x+3=5$ 5. $3+x=5$ 7. $x+7=15$
4. $x-8=12$ 6. $20+x=24$ 8. $x-10=20$

9. Add 2 to both members of the equation $x+3=5$. (*Ans.* $x+3+2=5+2$, or $x+5=7$.) Solve $x+5=7$ and compare the result with the solution of $x+3=5$.

10. Subtract 2 from both members of the equation $x+3=5$. (*Ans.* $x+3-2=5-2$, or $x+1=3$.) Solve $x+1=3$ and compare with the solution of $x+3=5$.

The same number may be added to, or subtracted from, both members of an equation without changing the value of the unknown number.

If $x=4$, $4\times x$, or $4x$, $=4\times 4=16$.
If $4x=16$, $\frac{1}{4}$ of $4x$, or x, $=\frac{1}{4}$ of 16, or 4.

Both members of an equation may be multiplied, or divided by, the same number without changing the value of the unknown number.

WRITTEN

Find the value of x in the following equations:

11. $5x=25$ 13. $4x+x=45$ 15. $2x+3x=35$
12. $6x=36$ 14. $5x-x=24$ 16. $9x-5x=12$

17. If $\frac{1}{3}$ of x, or $\frac{x}{3}$, equals 12, how much does x equal? (*Ans.* 3×12, or 36.) If $\frac{2}{3}$ of x, or $\frac{2x}{3}$, equals 12, how much does x equal? (*Ans.* $\frac{1}{2}$ of 3×12, or $\frac{1}{2}$ of 12, or 18.)

18. If $\frac{2x}{3} = 3$, find the value of x.

SOLUTION. Since 3 times $\frac{2x}{3}$ equals $2x$, multiply both members by 3 to make the first member an integer, $2x = 9$. Then divide both members by 2. *Ans.* $x = 4\frac{1}{2}$.

Solve the following equations:

19. $\frac{x}{2} = 10$ **21.** $\frac{3x}{5} = 15$ **23.** $\frac{3x}{4} = 9$

20. $\frac{2x}{3} = 8$ **22.** $\frac{4x}{9} = 16$ **24.** $\frac{5x}{7} = 25$

PROBLEMS

WRITTEN

In every problem certain statements, or suppositions, are given, and a question asks for a number whose value is unknown. The finding of this unknown number requires the making and solving of an equation in which the unknown number is represented by a letter.

The first step is to let x, or some other letter, represent the unknown number. This letter may then be used whenever it is necessary to employ the unknown number in the solution of the problem.

1. If a number is increased by 250, the result is 3296. What is the number?

SOLUTION

Let x = the number

Then $x + 250 = 3296$.

$x = 3296 - 250$.

$x = 3046$.

PROOF. $3046 + 250 = 3296$.

2. If 295 is subtracted from a number, the result is 488. What is the number?

3. Ella's age is 12 years less than Frank's, who is 25 years old. How old is Ella?

4. Eva and Ruth together have $54. Eva has twice as much as Ruth. How much has each?

SOLUTION

Let x = the number of dollars Ruth has.
Then $2x$ = the number of dollars Eva has,
and $x + 2x = 54$.
Hence, $3x = 54$.
$x = 18$, the number of dollars Ruth has.
$2x = 2 \times 18 = 36$, the number of dollars Eva has.
PROOF. 36 = 2 times 18; 36 + 18 = 54.

5. Separate the number 600 into two parts, one of which shall be 5 times the other.

6. A horse and carriage cost $274. Find the cost of each if twice the cost of the carriage is $34 less than the cost of the horse.

7. If $\frac{3}{5}$ of a number is 72, what is the number?

8. What number added to 3 times itself equals 240?

9. If 6 times a number plus 15 equals 315, what is the number?

10. If 5 times a number less 48 equals 302, what is the number?

11. If a number is increased by 3 times itself, the result will be 72. What is the number?

12. A school in which there are 500 pupils has 56 more girls than boys. How many of each are there?

13. Find the interest of \$1000 at 6% for 5 yr., using the formula $i = p \times r \times t$.

SOLUTION. Substitute \$1000 for p, 6%, or $\frac{6}{100}$, for r, 5 for t

$$\$\cancel{1000} \times \frac{6}{\cancel{100}} \times 5 = \$300.$$

14. Form five problems in interest and solve them by the above formula.

15. Find the area of a rectangle having a base of 6 in. and a height, or altitude, of 8 in., using the formula, $A = b \times h$.

$A = b \times h$. $A = 4 \times 8 = 32$. *Ans.* 32 sq. in.

16. Find the area of a rectangle having a base of 6 in. and an altitude of 10 in., using the above formula.

17. Form five problems like 15 and 16, and solve them by the use of the formula given above.

18. Three men, A, B, and C, formed a partnership with \$30,000 capital. B's capital was 3 times A's, and C's as much as the difference between B's and A's. How much capital did each furnish?

19. A girl made three purchases which amounted to \$27. For shoes she paid \$2 more than for gloves, and for a dress \$17 more than for shoes. How much did each purchase amount to?

20. The sum of three numbers is 24. The second is 5 less than the first, and the third is 10 more than the second. What are the numbers?

21. Divide the number 33 into three parts such that the second shall be 3 less than the first, and the third 4 times the first.

22. What must be added to 15 to make 42? What must be added to x to make 42?

23. Divide 18 into two parts, the smaller of which is x. What is the other part? The smaller part equals $\frac{1}{2}$ the larger part. Find each part.

24. A pipe will fill a tank in 8 hours. What part of the tank will it fill in 1 hour? in 3 hours? A pipe will fill a tank in b hours. What part of the tank will it fill in 1 hour? in c hours?

25. 60 is 75% of what number?

26. Divide the number 57 into three parts such that the second shall be 4 more than the first and the third 8 less than the second.

27. If a number is increased by 7 times itself and 40, the result is 360. What is the number?

28. Separate the number 240 into three parts, the second of which is 3 times the first and the third 4 times the first.

29. What number diminished by 50 equals 712?

30. What number divided by 17 equals 20?

31. What number multiplied by 25 equals 625?

32. What number increased by 83 equals 214?

33. What number decreased by 20% of itself equals 40?

34. What number increased by 20% of itself equals 30?

35. Separate 100 into two parts, the smaller of which, x, equals $\frac{1}{3}$ of the larger part.

36. A mother is 4 times as old as her daughter, and the sum of their ages is 40 years. How old is each?

37. The sum of the three numbers is 30. If the second number is twice the first, and the third number is 3 times the first, what is each number?

38. Construct and solve six problems, one each like problems 29–34 above.

POWERS AND ROOTS

STUDY RECITATION

A **power** is the product arising from using the same number two or more times as a factor.

$3 \times 3 = 3^2 = 9$.

9 is the product of 3 used twice as a factor.

In 3^2 the figure 2 shows how many times 3 is to be used as a factor.

If a number is used twice as a factor, the product is the **square**, or the **second power**, of the number.

9 is the square of 3, or 9 is the second power of 3.

If a number is used 3 times as a factor, the product is the **cube**, or the **third power**, of the number.

$4 \times 4 \times 4 = 4^3 = 64$. The 3 in 4^3 shows that 64 is the product of 4 used 3 times as a factor. 64 is called the cube of 4, or the third power of 4.

A number written at the right of, and a little above, another number is called an **exponent**, and shows how many times the other number is to be used as a factor.

In the expressions 5^2, 7^3, 8^4, 9^6, the exponents are 2, 3, 4, and 6. The expressions are read, 5 square, or the second power of 5, or 5 to the second power; 7 cube, or the third power of 7, or 7 to the third power; the fourth power of 8, or 8 to the fourth power; the sixth power of 9, or 9 to the sixth power.

What does each of these expressions mean? Write the equivalent of each expression in two ways.

To the Teacher. A number is sometimes called the first power of itself; as, 2 is the first power of 2; but as there must be two factors in order to have a product, and as a power is the product of equal factors and 2 is not such a product, it cannot properly be called a power of itself. In the higher mathematics, however, the expression *first power* is used.

Read the following expressions. Give the value of each at sight:

1. 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 20^2, 50^2.

Learn this table:

$13^2 = 169$	$16^2 = 256$	$19^2 = 361$	$23^2 = 529$
$14^2 = 196$	$17^2 = 289$	$21^2 = 441$	$24^2 = 576$
$15^2 = 225$	$18^2 = 324$	$22^2 = 484$	$25^2 = 625$

To find a power of a fraction, reduce it to its lowest terms, and find the required power of each of its terms.

Find the powers indicated in the expressions below:

2. $26^2 = ?$ $28^2 = ?$ $48^2 = ?$ $55^2 = ?$ $75^2 = ?$ $100^2 = ?$
3. $8^3 = ?$ $7^3 = ?$ $9^3 = ?$ $10^3 = ?$ $100^3 = ?$ $110^3 = ?$
4. $(\frac{1}{2})^2 = ?$ $(\frac{2}{3})^2 = ?$ $(\frac{4}{5})^2 = ?$ $(\frac{3}{8})^2 = ?$ $(\frac{5}{6})^2 = ?$ $(\frac{9}{12})^2 = ?$
5. $.1^2 = ?$ $.5^2 = ?$ $.01^2 = ?$ $.15^2 = ?$ $.001^2 = ?$ $.99^2 = ?$

Involution is the operation of finding the product of equal factors, or of raising a number to a power.

A **root** of a number is one of the *equal* factors of the number. If it is one of **two** equal factors of a number, it is the **square root** of the number.

$6^2 = 36$. 36 is the square of 6, or the second power of 6. 6 is one of the two *equal* factors of 36, and is therefore the square root of 36.

$5^3 = 125$. 125 is the cube of 5, or the third power of 5. 5 is one of the three *equal* factors of 125, and is therefore the cube root of 125.

In the expression $5^3 = 125$, 125 is the power, 5 is the root of the power, and 3, the exponent, shows to what power 5 is to be raised.

Evolution is the operation of finding the equal factors, or the roots of a power.

A root is indicated by the **radical sign** $\sqrt{\ }$, placed over the number whose root is to be found. $\sqrt{25}$ indicates that the square root of 25 is to be found. The figure 2 is not written with the sign; but for cube root, fourth root, etc., a figure called the **index** is placed before the sign to tell what root is to be found. $\sqrt[3]{125}$ indicates the cube root of 125. $\sqrt[4]{81}$ indicates the fourth root of 81.

STUDY RECITATION

1. Square 25. $25 = 20 + 5$, hence it may be squared in two ways:

$25 =$		$20 + 5$	
$25 =$		$20 + 5$	
$125 =$	$20 \times 5 + 5^2$		(Product of 20 + 5 by 5)
$500 =$	$20^2 + 20 \times 5$		(Product of 20 + 5 by 20)
$625 =$	$20^2 + 2(20 \times 5) + 5^2$		(Product of 20 + 5 by 20 + 5)

The square of any integer composed of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

WRITTEN

Square the following:

2. 15	**4.** 35	**6.** 16	**8.** 27	**10.** 150
3. 25	**5.** 40	**7.** 19	**9.** 84	**11.** 250

The **squares** of the smallest and the largest integers composed of one, two, and three figures are as follows:

$1^2 = 1$	$10^2 = 100$	$100^2 = 10000$
$9^2 = 81$	$99^2 = 9801$	$999^2 = 998001$

12. Separate each of these squares into periods of two figures each, beginning at the right. Thus, 98′01, 1′00′00, 99′80′01.

13. How does the *number of periods* in each square compare with the *number of figures* in the corresponding roots?

The number of figures in the square root of a perfect square is the same as the number of periods of two figures each into which the number can be separated, beginning at units.

The left-hand period may contain only one figure.

ROOTS BY FACTORING

STUDY RECITATION

Since $5 \times 5 = 25$, $\sqrt{25} = \sqrt{5 \times 5} = 5$.

Since $5 \times 5 \times 5 = 125$, $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$.

1. Find, by factoring, the square root of 900.

SOLUTION. The prime factors of 900 are $3 \times 3 \times 2 \times 2 \times 5 \times 5$.
Arranging these in two equal groups, you have $(2 \times 3 \times 5) \times (2 \times 3 \times 5)$.
Hence $\sqrt{900} = 2 \times 3 \times 5$, or 30.

2. Find, by factoring, the cube root of 216.

SOLUTION. The prime factors of 216 are $2 \times 2 \times 2 \times 3 \times 3 \times 3$.
Arranging these in three equal groups, you have $(2 \times 3)(2 \times 3)(2 \times 3)$.
Hence $\sqrt[3]{216} = 2 \times 3$, or 6.

Find, by factoring, the square root of:

	a.	*b.*	*c.*	*d.*	*e.*
3.	49	81	144	729	2500
4.	64	121	361	324	3600

Find, by factoring, the cube root of:

5.	27	125	343	729	4096
6.	64	216	512	1000	3375

SQUARE ROOT

STUDY RECITATION

1. Extract the square root of 625.

```
  6'25(25
    4
45 | 2 25
   | 2 25
```

Steps:

1. Separate the number (625) into periods of two figures each, beginning with units.

2. Find the greatest square (4) equal to or less than the left-hand period (6), write its square root (2) for the first figure in the root, and subtract the square (4) from the left-hand period (6).

3. To the remainder (2) annex the next period (25) for a dividend.

4. Double the root (2) already found, and write the product (4) at the left of the remainder for a trial divisor.

5. Divide the new dividend (225), exclusive of the right-hand figure (5) by the trial divisor (4); write the quotient (5) for the next figure of the root, and also at the right of the trial divisor, to make the complete divisor (45).

6. Multiply the complete divisor (45) by the last figure (5) of the root, and subtract the product (225) from the dividend.

7. If there are other periods remaining, proceed in the same way.

NOTE. The seven steps should be *memorized*, omitting the numbers, as they apply to this example only.

2. Extract the square root of 1296.

Follow the directions above, without regarding the numbers.

```
  12'96(36
    9
   | 3
   | 3 96
6  | 3 96
66 | 3 96
   | 3 96
```

Step 1. State it. What is the first period at the left?

Step 2. State it. What is the greatest square less than 12?

Step 3. State it. What is the new dividend?

Step 4. State it. What is the trial divisor?

Step 5. State it. What is the new dividend, exclusive of the right-hand figure? 6 is contained in 39 how many times? Where is the quotient placed?

Step 6. State it.

Bringing the work together (as in example 1) to shorten the operation, you have the following:

```
12'96(36
  9
66 | 3 96
     3 96
```

3. Extract the square root of 60,516.

```
6'05'16(246
  4
44 | 2 05
     1 76
486 | 29 16
      29 16
```

In the fifth step, on dividing 20 by the trial divisor 4, you get 5; but when 5 is annexed to the trial divisor and the complete divisor 45 is multiplied by 5, the product is found to be larger than the dividend. This shows that the quotient 5 is too large, and must be decreased by 1, making it 4. Bring down the next period and proceed as before, doubling the number already found in the root, and dividing 291 by 48, etc. If there were still other periods, the process would be continued in the same way.

4. Extract the square root of 94,249.

```
9'42'49(307
  9
607 | 4249
      4249
```

The trial divisor 6 is not contained in 4. In such cases, place a zero in the root and at the right of the trial divisor, and bring down the next period. Divide 424 by 60; place the quotient 7 in the root and at the right of the trial divisor.

If, after annexing a zero as above and bringing down the next period, the trial divisor is not contained in the dividend, exclusive of the right-hand figure, annex another zero, bring down another period, etc.

When both terms of a fraction whose square root is to be extracted are squares, extract the square root of each. Otherwise, reduce the fraction to a decimal, carrying on the division until the decimal has twice as many places as there are figures desired in the root. Then extract the square root of the decimal.

In extracting the square root of decimals, begin at the decimal point and point off to the right into periods of two places, annexing zeros if necessary.

5. Point off .47856.

.47′85′60 Begin at the decimal point and point off to the right, annexing one zero, so that each period shall have two places; thus, .47′85′60.

In pointing off mixed decimals, point from the decimal point to the left in the integral part and to the right in the decimal part, annexing a zero if necessary to make the last period in the decimal two places.

A **perfect square** is the product of two equal factors.

When the number whose root is to be extracted is not a perfect square, the root can be found approximately, but not exactly, by annexing periods of zeros after the decimal point, carrying the work as far as desired. It is usually sufficient to carry the work out two decimal places.

6. Extract the square root of 295.

7. Extract the square root of 6.

2′95.′00′00(17.17+		6.′00′00′00(2.449	
1		4	
27 \| 195	189	44 \| 200	176
341 \| 600	341	484 \| 2400	1936
3427 \| 25900	23989	4889 \| 46400	44001

WRITTEN

Find the square root of:

8.	2304	**15.**	1156	**22.**	492804	**29.**	39.8161
9.	2025	**16.**	5184	**23.**	256036	**30.**	50.8369*
10.	256	**17.**	133225	**24.**	62500	**31.**	66.2596
11.	4225	**18.**	104976	**25.**	490000	**32.**	*345
12.	7225	**19.**	522729	**26.**	45.1584	**33.**	*5
13.	4096	**20.**	98596	**27.**	3003.04	**34.**	$\frac{16}{25}$
14.	729	**21.**	390625	**28.**	27.5625	**35.**	$\frac{9}{16}$.

NOTE. For further explanation of the process of square root, see Supplement, p. 485.

* To 2 decimal places.

PROBLEMS

WRITTEN

1. A square piece of land contains 1440 A. How many rods long and wide is it?

2. What is the cost of fencing a square farm of 160 A. at the rate of $4 per rod?

3. At 60¢ a rod, what is the cost of fencing a square farm whose area is 262 A. 105 sq. rd.? How much additional will it cost to divide it into 3 equal parts by building cross fences?

4. A farmer wishes to fence a field in the form of a rectangle twice as long as it is wide, and containing 20 A. How many rods of fence are required?

How many more rods of fence are necessary to divide the field into two square fields? Draw a figure to represent the rectangle divided into two equal squares.

5. Having the area of a rectangle twice as long as it is wide, how can you find the length of the sides?

6. A hallway having an area of 108 sq. ft. is three times as long as it is wide. What is its width and length?

SUGGESTION. 108 sq. ft. ÷ 3 = area of each of the three squares into which the rectangle may be divided.

7. A square 10-acre field is planted with corn. The hills are four feet apart each way. The outside hills are on the extreme edge of the field on each side. How many hills of corn are there in the field?

8. A strawberry bed has an area of 2592 sq. ft. It is twice as long as it is wide. Find its dimensions in rods and the fractions of a rod.

REVIEW

1. Define power; square; cube; exponent; involution.

2. What is meant by evolution in arithmetic? by square root?

3. What is the difference between a root and a factor of a number?

4. What does the radical sign indicate? What is the index of a root?

5. What is the relation of the number of figures in a root to the number of figures in its square?

6. What is a trial divisor? How is it found? On what principle?

7. Extract the square root of 5329 and explain each step in the operation.

Problems without Numbers

8. How can you find the square of a number consisting of tens and units?

9. How can you find the square root of a number by factoring?

10. How can you find the cube root of a number by factoring?

11. Give the rule for the extraction of square root.

12. How is the square root of a common fraction extracted? the square root of a decimal?

13. When a number is not a perfect square, how may its approximate square root be found?

14. Make a problem demanding for its solution the extraction of a square root. Solve it.

MENSURATION

Mensuration treats of the measurements of lines, angles, surfaces, and solids.

A **plane surface** or a **plane figure** is a surface such that a straight line joining any two parts of it lies wholly in the surface.

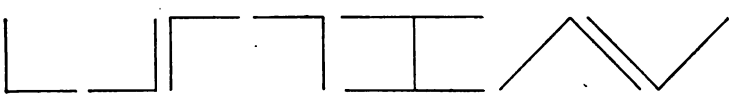
An **angle** is the difference in direction of two lines that meet.



RIGHT ANGLE ACUTE ANGLE OBTUSE ANGLE

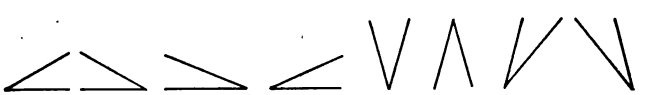
Two straight lines that meet so as to form a square corner make a **right angle.** The lines are said to be **perpendicular** to each other. If one line is drawn perpendicular to another, not at its extremity, two equal angles are formed. Each of these angles is a right angle.

All right angles are equal.



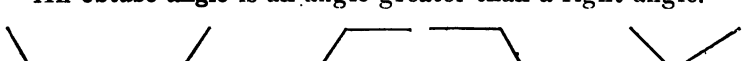
RIGHT ANGLES

An **acute angle** is an angle less than a right angle.



ACUTE ANGLES

An **obtuse angle** is an angle greater than a right angle.



OBTUSE ANGLES

QUADRILATERALS

A **quadrilateral** is a plane figure having four sides.

Parallel lines are lines that are everywhere equally distant from each other.

A **parallelogram** is a quadrilateral whose opposite sides are parallel.

A **rectangle** is a parallelogram whose angles are all right angles.

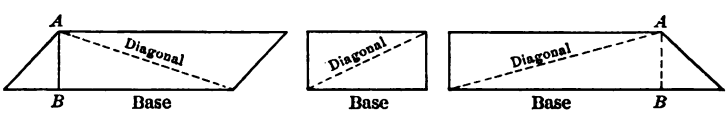
A **square** is a rectangle whose sides are equal.

A **trapezoid** is a quadrilateral having only two sides parallel.

Any line (AB below) perpendicular to and connecting the parallel sides of a parallelogram or of a trapezoid is called the **altitude**.

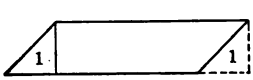
The **diagonal** of a quadrilateral is a straight line joining the vertices of opposite angles.

The line on which the parallelogram is supposed to stand is called the **base**. The opposite side may also be regarded as a base.



PARALLELOGRAM RECTANGLE TRAPEZOID

TO THE TEACHER. Teach the definitions by means of blackboard drawings, figures cut from pasteboard, etc. When the definitions have been taught, test the pupil's knowledge thereof by requesting him to draw the figures you designate by name.

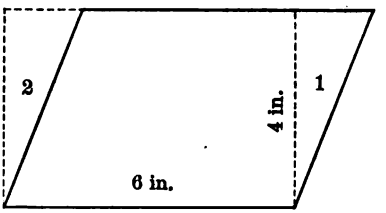


Prove by means of cardboard figures similar to the accompanying one the rules for finding the areas of plane figures, and the reasons for the rules.

STUDY RECITATION

Define square, quadrilateral, parallelogram, rectangle. What is the difference between a square and a parallelogram? between a quadrilateral and a parallelogram?

Cut from paper or pasteboard a rectangular parallelogram 6 inches long and 4 inches high. Cut another parallelogram of the same size, similar in shape to the accompanying figure. What is the area of the rectangular parallelogram? Cut off the projecting part marked "1," and put it at the other end of the figure, marking it "2." Compare the area of the parallelogram thus formed with the preceding one. What is its area?



Any parallelogram has the same area as a rectangle having the same base and altitude.

To find the area of a parallelogram, multiply the base by the altitude.

NOTE. In this and similar rules, by *base* and *altitude*, or other dimensions, is meant the numbers representing them, when expressed in *like units*. In these cases, the product represents the number of surface units in the area, each surface unit being the square of one of the linear units of the base or the altitude.

PROBLEMS

ORAL

1. What is the area of a parallelogram 10 in. wide and 15 in. long?

2. How many acres are there in a piece of land 80 rd. long and 60 rd. wide?

3. A field is 80 rd. long and 40 rd. wide. How long a fence will be required to divide the field into two equal squares? How many acres are there in each square?

Written

4. What is the area of a parallelogram having a base of 15 in. and an altitude of $9\frac{1}{2}$ in.?

5. A strip of land in the shape of an oblique parallelogram is 150 rd. long and $60\frac{2}{3}$ rd. wide. How many acres does it contain?

6. What is the difference in area of two strips of land in the shape of parallelograms, one having a 40-rd. base and 165-ft. altitude, and the other a 150-yd. base and 181.5-ft. altitude?

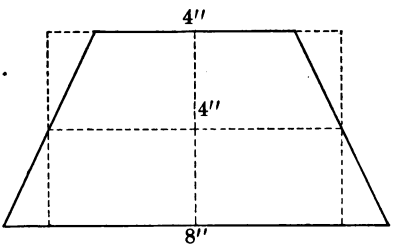
7. The area of a parallelogram is 650 sq. ft., and the base is 40 ft. Find the altitude.

8. The altitude of a parallelogram that has an area of 36 sq. yd. is 8 ft. What is the length of the base?

9. Given the area of a parallelogram and either the base or the altitude, how can you find the other dimension?

Study Recitation

What is a trapezoid?



Construct a trapezoid 4 in. high, with an 8-in. base and a 4-in. top. Halfway between the bases draw a line parallel to them. Cut off the corners of the trapezoid, and place them as shown in the illustration. What kind of figure have you? What are its dimensions? What is its area? Solve the example by the rule for finding the area of a trapezoid, and compare the result with the preceding answer.

To find the area of a trapezoid, multiply one half the sum of the parallel sides by the altitude.

PROBLEMS

WRITTEN

1. Find the area of a trapezoid whose sides are 12 in. and 18 in., the perpendicular distance being 10 in.

2. Find the area of a trapezoid, one side of which is 125 in. and the other 96 in., the altitude being 48 in.

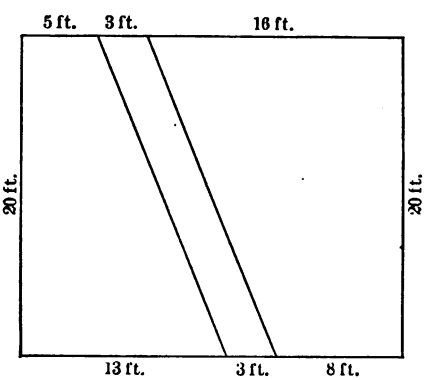
3. What is the area of a plank $16\frac{1}{2}$ in. on one end, $8\frac{3}{4}$ in. on the other, and 10 ft. long?

NOTE. First make a mental-estimate, omitting the fractions.

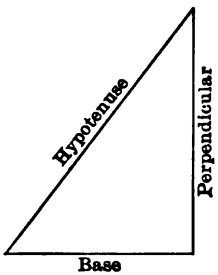
4. A has a field in the form of a trapezoid. The parallel sides are 75 rd. and 95 rd., and the perpendicular distance between them is 66 rd. How many acres are there in the field?

5. A board 18 ft. long is 12 in. wide at one end and 8 in. wide at the other. If the board is $1\frac{1}{2}$ in. thick, how many board feet does it contain?

6. The accompanying figure represents a garden with a 3 ft. walk running across it as shown. Using the dimensions given, find the area of the part of the garden on each side of the walk. Find the area of the entire plot. Find the area of the walk. See whether the sum of the areas of the walk and the parts of the garden on both sides of the walk equals the area of the entire plot.



TRIANGLES

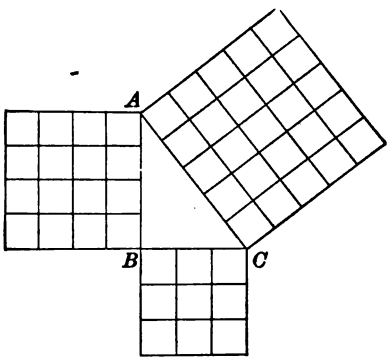


A triangle is a plane figure having three sides.

A triangle having a right angle is called a **right-angled triangle.**

The **hypotenuse** is the side opposite the right angle. The other two sides are the **base** and the **perpendicular.**

Right-angled triangles.



STUDY RECITATION

1. How many small squares are there in the large square on side AB of the triangle? on side BC? on side AC?

2. Compare the number of squares on the hypotenuse with the sum of the squares on the other two sides.

To find one side of a right-angled triangle when the other sides are given, square the two sides. If the hypotenuse is not given, extract the square root of the sum of the two squares to find the hypotenuse. If the hypotenuse is given, extract the square root of the difference of the squares to find the required side.

3. If the given sides are 4 in. and 3 in., find the hypotenuse.

SOLUTION

$4^2 + 3^2 = 16 + 9 = 25$; $\sqrt{25} = 5$. *Ans.* 5 in.

4. If the hypotenuse is 5 in. and one side is 3 in., find the other side.

SOLUTION

$5^2 - 3^2 = 25 - 9 = 16$; $\sqrt{16} = 4$. *Ans.* 4 in.

PROBLEMS

WRITTEN

1. The two sides of a right-angled triangle are 51 ft. and 68 ft. respectively. What is the length of the hypotenuse?

SOLUTION. $\sqrt{51^2 + 68^2} = \sqrt{7225} = 85$, number of feet.

2. The hypotenuse of a right-angled triangle is 115 in., and the base is 92 in. What is the perpendicular?

3. A ladder 65 ft. long is placed against a house on level ground so that it reaches a window 60 ft. from the ground. How far from the house is the foot of the ladder?

4. One vessel sails due north from a port at the rate of 3 miles an hour. Another, sailing at the same time, goes east at the rate of 4 miles an hour. How far are they apart in 2 days?

5. A road runs on two sides of a square 40-acre field. In going from one corner to the diagonally opposite corner, how much will be saved by going in a straight line rather than by following the road?

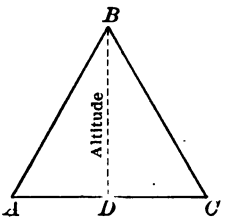
6. A smoke stack has guy wires anchored at points 80 ft. distant on a horizontal line from the base of the chimney. The wires are 120 ft. long. At what height above the base of the chimney are the wires attached to it? Give the answer in the nearest whole number of feet.

7. A triangular sail measures 9 ft. on the base and 12 ft. on the perpendicular. What is the perimeter of the sail?

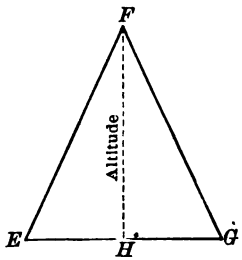
8. If a ladder 65 ft. long leans against a house and the foot of the ladder is 25 ft. from the base of the house, find the height of the top of the ladder from the ground.

Other triangles.

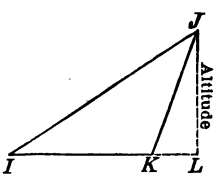
A triangle with three equal sides is an **equilateral triangle.** The angle opposite the base of a triangle is the **vertex.**



EQUILATERAL TRIANGLE



ISOSCELES TRIANGLE



SCALENE TRIANGLE

The **altitude** of a triangle is the length of a perpendicular line from the vertex to the base, or to the base extended.

A triangle having two sides equal is an **isosceles triangle.**

A triangle having no two sides equal is a **scalene triangle.**

The altitude of the equilateral or of the isosceles triangle divides the triangle into two equal right-angled triangles. Half the base of either of the original triangles is the base of one of the two equal triangles formed from it. If the base and another side of an equilateral, or of an isosceles, triangle are known, its altitude and area may be found.

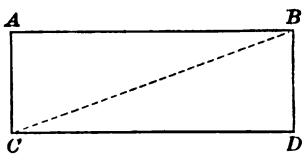
In a scalene triangle, a line may be drawn from one of its angles, regarded as the vertex of the triangle, to the opposite side and perpendicular to that side, dividing the triangle into two right-angled triangles. If the length of this perpendicular and the length of another side of each of the two triangles are known, their areas can be found. The sum of the areas equals the area of the original triangle.

PROBLEMS

Oral

1. What is the difference between a right-angled triangle and an equilateral triangle? between an equilateral triangle and an isosceles triangle? between a scalene triangle and an isosceles triangle?

2. Construct a parallelogram 6 in. long and 4 in. high. What is its area?



3. Cut the parallelogram in two on a line joining two opposite corners. Compare the two triangles produced. What is the area of each? What is the length of each? the height?

4. Draw rectangles of different dimensions.

5. Draw a diagonal in each rectangle and compute the area of the rectangle and of each triangle.

6. What is the area of a triangle with a 10-in. base and 12 in. high? of a triangle with a 6-in. base and 8 in. high?

7. How many acres are there in a triangular field having a base of 320 rd. and an altitude of 60 rd.?

To find the area of a triangle, multiply the base by one half of the altitude.

Written

8. What is the area of a triangle whose base is 75 rd. and altitude 81 rd.?

9. A triangle has a $17\frac{1}{2}$-in. base and a 20-in. altitude. What is its area?

10. What is the area of a scalene triangle with a 30-in. base and a $1\frac{1}{2}$-yd. altitude?

Study Recitation

To find the area of a triangle when its three sides are known, find half the sum of the three sides; subtract each side from this number; find the product of half the sum and the three differences; find the square root of this product.

1. The sides of a triangle are 6, 8, and 10.

Solution

$\frac{1}{2}(6+8+10)=12$; $12-6=6$; $12-8=4$; $12-10=2$.
$12\times 6\times 4\times 2=576$; $\sqrt{576}=24$, area of the triangle.

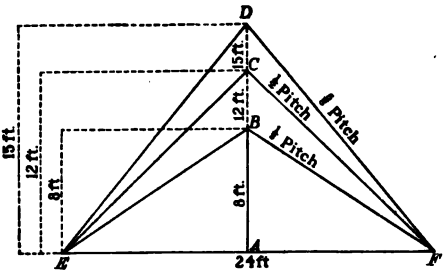
Written

2. Find the area of a triangular piece of land whose sides measure 30 rd., 20 rd., and 40 rd., respectively.

3. The sides of a triangular piece of linoleum measure 9 in., 12 in., and 15 in., respectively. What is its area? This area is what part of a square foot?

Find the area of the triangles, the lengths of whose sides are given below:

	a.	*b.*	*c.*		*a.*	*b.*	*c.*
4.	12 ft.	16 ft.	24 ft.	6.	14 yd.	12 yd.	18 yd.
5.	15 rd.	25 rd.	30 rd.	7.	7 in.	12 in.	16 in.



Pitch of roofs.

In the figure on p. 418 let the lines *EBF*, *ECF*, and *EDF* represent the front edges of three roofs, respectively, *EF*, a horizontal line, representing the distance between the upper outside edges of the plates on which the rafters rest.

The line *EF* represents the width of the building, 24 ft.

The line *AB* is the perpendicular distance from the peak to the middle of *EF*.

The roof *EBF* has a $\frac{1}{2}$ pitch.

The height of the peak from the line connecting the top of the plates is $\frac{1}{2}$ the distance from *E* to *F*. $\frac{1}{2}$ of 24 ft. = 8 ft., length of *AB*.

The roof *ECF* has a $\frac{1}{2}$ pitch.

The height of the peak from the line *EF* is $\frac{1}{2}$ *EF*. $\frac{1}{2}$ of 24 ft. = 12 ft., length of *AC*.

The roof *EDF* has a $\frac{5}{8}$ pitch.

The height of the peak from the line *EF* is $\frac{5}{8}$ *EF*. $\frac{5}{8}$ of 24 ft. = 15 ft., length of *AD*.

BAF, *CAF*, and *DAF* are right-angled triangles.

PROBLEMS

WRITTEN

1. Find the length of the rafters in each roof, from the peak to the outer edge of the plate. If the rafters extend 1 ft. beyond the plate, what is their length in each roof? What is the width of the roof on each side of the ridge?

2. If the building is 36 ft. long, and the ends of the roof project 1 ft. beyond the end wall, what is the length of the roof?

3. Using the dimensions found in the preceding problems, find the area of each roof.

4. How many bunches of shingles, 250 in each bunch, will be required to cover each roof, allowing 1000 to a square?

5. How can you find the pitch of the roof when the height of the peak above the top of the plates, and the width of the house, are known?

6. If the line EF (p. 418) is 30 ft. long and the line AB is 5 ft. long, what is the pitch of the roof?

7. If the line EF (p. 418) is 30 ft. long, and the roof ECF has a $\frac{1}{2}$ pitch, what is the length of the line AC?

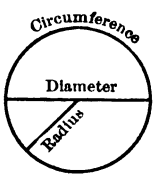
8. If the line EF is 30 ft. long and the line AD is $9\frac{3}{8}$ ft. long, what pitch has the roof?

Allowing for the same projection of the roof beyond the plates and at the ends of the building as in problems 1 and 2, and for 1000 shingles to a square, find the number of shingles required to shingle:

9. The roof indicated in problem 6.
10. The roof indicated in problem 7.
11. The roof indicated in problem 8.

CIRCLES

A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.



The **circumference** is the curved line that bounds the circle.

The **diameter** of a circle is a straight line passing through the center, with both ends terminating in the circumference.

The **radius** of a circle is a straight line from the center to the circumference. It is equal to half the diameter.

Define circle; diameter; circumference; radius.

By means of a piece of paper wrapped about a cylinder the diameter of which is known, find the circumference and determine how many times the diameter it is. It will be found to be a trifle more than $3\frac{1}{7}$ times the diameter.

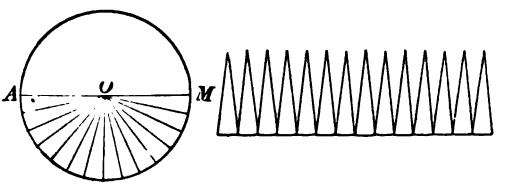
The circumference of a circle is more exactly 3.1416 times the length of its diameter. 3.1416 is often represented by the symbol π, called **pi**.

STUDY RECITATION

To find the circumference of a circle when the diameter is known, multiply the diameter by 3.1416.

1. What is the circumference of a circle having a 12-inch diameter? a 5-inch radius?

2. Construct a circle on pasteboard and cut it out. Divide one half of the circle into smaller sections as shown in the figure.



3. Cut the circle into two equal parts along the line AM. From O cut along the lines indicated toward the circumference, but do not separate the parts entirely. Spread the resulting figures out and combine them as indicated in the figure at the right.

If the sections cut out are small enough, each may be regarded as a triangle whose base is a straight line, and a small part of the circumference.

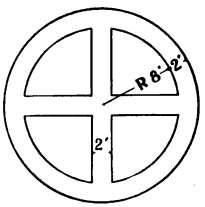
What is the length of the combined bases of the triangles as compared with the circumference of the circle? of the height of the triangles as compared with the radius? How is the area of the figure found?

To find the area of a circle, multiply the circumference by one half of the radius. Or, multiply the square of the radius by 3.1416.

WRITTEN

4. What is the area of a circle 10 feet in diameter? What is the area of a circle having a 20-foot radius?

5. Over how many square feet of surface can a horse graze if it is fastened to a post with a rope that allows it to graze 100 feet from the post?



6. A circular garden having a radius of 8′ is divided into quadrants by cross walks 2′ wide. It is also bounded by a circular walk 2′ wide. How many bricks are required to lay the walks when the bricks, $2\frac{1}{4}''$ by $4''$ by $8\frac{1}{4}''$, are laid flat? Regard the cross walks as rectangles 16′ long and 2′ wide. (Allow 39 bricks to 1 sq. yd.)

Indicating Radius by R, Diameter by D, Circumference by C, and Area by A, the two rules given on p. 421 may be expressed by formulas 1, 2, and 3 below.

Formulas.

1. $D \times 3.1416 = C.$
2. $C \times \frac{R}{2} = A.$
3. $R^2 \times 3.1416 = A.$

Formula 3 is derived from formula 2 by substituting for C its value as given in formula 1, and substituting $2 \times R$ for its equal D, $2 \times R \times 3.1416 \times \frac{R}{2} = A$. Canceling the factor 2, and multiplying R by R, you get formula 3, $R^2 \times 3.1416 = A$.

Since the square of any number is $\frac{1}{4}$ the square of twice the number, the square of the radius is $\frac{1}{4}$ the square of the diameter, or $R^2 = \frac{D^2}{4}$. Substituting $\frac{D^2}{4}$ for R^2 in formula 3, and canceling the factor 4, you get,

4. $D^2 \times .7854 = A.$

From the foregoing formulas, other formulas may be derived by applying the principle: *If the product of two factors is divided by either factor, the quotient is the other factor.*

		Derived from Formula	
5.	$\frac{C}{3.1416} = D,$	1.	Read: Circumference divided by 3.1416, equals the diameter.

		Derived from Formula	
6.	$\frac{A}{C}=\frac{R}{2}$,	2.	Read: Area divided by circumference equals one half the radius.
7.	$\frac{A}{\frac{R}{2}}=C$,	2.	Read: Area divided by one half the radius equals the circumference.
8.	$\frac{A}{3.1416}=R^2$,	3.	Read: Area divided by 3.1416 equals the square of the radius.

Formulas 1 and 2 and 3 should be thoroughly memorized; the remaining formulas may be derived from these when needed.

9.	$\frac{A}{.7854}=D^2$.	4.	Read: Area divided by .7854 equals the square of the diameter.

Find the required element in the following exercises and express the formula employed for finding it:

7.	$D=100$	$C=?$	13.	$A=50.2656$	$R=?$
8.	$C=628.32$	$D=?$	14.	$A=50.2656$	$C=?$
9.	$R=7$	$C=?$	15.	$A=1$ acre	$D=?$
10.	$R=5$	$A=?$	16.	$C=157.08$	$A=?$
11.	$D=120$	$A=?$	17.	$R=31$ ft. 6 in.	$A=?$
12.	$A=218.25$	$D=?$	18.	$D=10$	$A=?$

19. The area of a circular garden is 2578.23 sq. rd. Find the diameter and the circumference.

20. The circumference of a circle is 15.708 ft. What is its area? How many circles, each 15 in. in diameter, will equal the area of the above circle?

21. A hot air flue is 24 in. by 36 in. What is the diameter of a circular flue having the same area?

22. What is the difference in area of two flues, one 16 in. square, and the other a circular flue 16 in. in diameter?

23. Draw a figure to represent an 8-in. square and circumscribe a circle about it. What is the difference in the areas of the two figures?

24. The diameter of a water pipe is 3 in. What is the area of the cross section of the pipe?

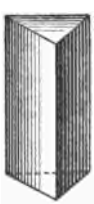
25. A circular silo has an inside diameter of 20 ft. How many square feet are there in the base of the silo?

SOLIDS

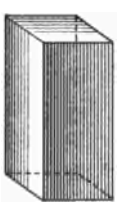
A **solid** or body has length, breadth, and thickness.

A **prism** is a solid whose sides are parallelograms, and whose two ends are equal polygons parallel to each other.

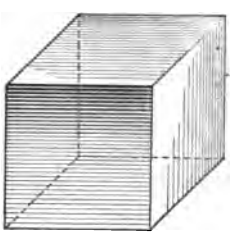
Prisms, according to the form of their bases, are triangular, quadrangular, pentagonal, etc.



TRIANGULAR PRISM



QUADRANGULAR PRISM



CUBE

A **triangular** prism is one that has triangles for its bases.

A **quadrangular** prism is one that has quadrilaterals for its bases. Quadrangular prisms are also called **parallelopipeds.**

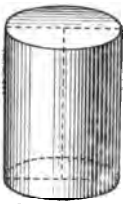
A **cube** is a solid with six equal square faces.

A **cylinder** is a solid with a curved surface and two equal parallel circular bases.

The **convex surface** of a cylinder is the area of its curved surface.

The **axis** of a cylinder is the line joining the center of the two bases.

The **altitude** of a prism or of a cylinder is the perpendicular distance between its bases.



CYLINDER

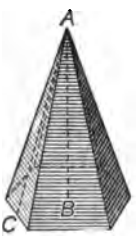
A **pyramid** is a solid whose base is a polygon, and whose sides are triangles meeting at the vertex.

The **altitude** of a pyramid is the distance from the vertex to the center of the base; as AB in the figure below.

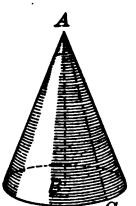
The **slant height** of a pyramid is the distance from the vertex to the middle of the base of any of its triangular sides; as AC in the figure.

A **cone** is a solid whose base is a circle and whose surface tapers uniformly to a point.

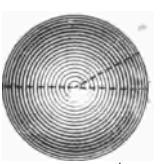
A **sphere** is a solid with a curved surface, every point of which is equally distant from a point within called the **center.**



PYRAMID



CONE



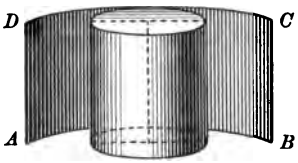
SPHERE

The **diameter** of a sphere is a straight line passing through the center of the sphere and terminating in its surface. One half the diameter is the **radius** of the sphere. The greatest distance around the sphere is its **circumference.**

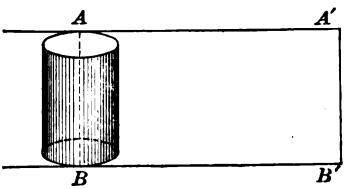
All the surface of a solid except its base or bases is called the **lateral surface.** The entire surface is the sum of the lateral surface and of the bases.

Study Recitation

Note. Derive the rules for finding the area of the surface of different solids in manner similar to the one that follows:



Cover exactly the lateral surface of any cylinder with a sheet of paper of the same height. Unroll the paper. Find its area. What is the circumference of the cylinder? Multiply the circumference of the cylinder by its height. How does the product compare with the area of the paper? State the rule for finding the lateral surface of a cylinder.



The cylinder in the illustration is represented as lying on a plane touching it along the line AB. Conceive it rolled to the right. When A reaches the plane at A', B is exactly at the point B', and the cylinder has made one rotation. The path described by it is a rectangle equal to the entire lateral surface of the cylinder.

The sides of any prism are rectangles. If these rectangles are placed side by side, they form one rectangle whose length is the perimeter of the base and whose width is the altitude of the prism.

The surface of a cylinder may be regarded as made up of an infinite number of rectangles, the sum of whose bases is the circumference of the cylinder. The length of each rectangle is the altitude of the cylinder.

The lateral surface of a pyramid is made up of triangles. The slant height of the pyramid is the altitude of each of the triangles. The perimeter of the base is the sum of the bases of the triangles.

A cone may be regarded as a pyramid with an infinite number of bases, therefore its lateral surface is made up of an infinite number of triangles whose altitude is the slant height of the cone and the sum of whose bases is the circumference of the base of the cone.

In finding the lateral surface of a prism, a cylinder, a pyramid or a cone, you simply find the area of three or more rectangles, or three or more triangles.

Surfaces of solids.

I. *The lateral surface of a prism or a cylinder equals the product of the perimeter or the circumference of the base by the altitude.*

II. *The lateral surface of a pyramid or a cone equals one half the product of the perimeter of the base by the slant height.*

III. *The surface of a sphere equals the product of the diameter by the circumference, or it equals the area of four times the circle having the diameter of the sphere.*

WRITTEN

1. What is the lateral surface of a cylinder 8 in. in circumference and 14 in. high? of a cylinder 10 in. in circumference and 10 in. high?

2. What is the lateral surface of a triangular prism whose sides are 3 in., 4 in., and 5 in., and whose height is 20 in.? What is the entire surface of this prism?

3. What is the lateral surface of a square prism whose base is 4 in. and altitude 9 in.? What is the entire surface?

4. What is the lateral surface of a cylinder with an altitude of 16 in. and a diameter of 12 in.? What is the entire surface?

5. What is the lateral surface of a cylinder having an altitude of 20 in. and a radius of 8 in.? the entire surface?

6. What is the lateral surface of a square pyramid with a 12-in. base and 20-in. slant height? the entire surface?

7. What is the lateral surface of a pyramid whose base is an equilateral triangle having 12-in. sides, and whose slant height is 18 in.?

8. What is the lateral surface of a cone 20 in. in diameter with a slant height of 40 in.? the total surface?

9. What is the surface of a 10-in. sphere?

10. What is the surface of a 20-in. sphere; of a 20-in. cube; of a circle 20 in. in diameter?

Volume of solids.

STUDY RECITATION

1. Define cube; parallelopiped; prism; pyramid; cone; sphere.

2. Draw or construct a cube 1 in. on each side. How many cubic inches does it contain?

3. Draw or construct a parallelopiped that has a 1-in. base and is 2 in. high. How many cubic inches does this solid contain? Construct it 3 in. high. How many cubic inches does it contain?

4. Draw or construct a prism or a parallelopiped 3 in. long, 2 in. wide, and 1 in. thick. How many cubic inches does it contain?

I. *The volume of a rectangular solid equals the product of its three dimensions, or it is equal to the product of the number representing the area of the base by the altitude.*

II. *The volume of a cylinder equals the product of the number representing the area of the base by the altitude.*

III. *The volume of a pyramid or a cone equals the product of the number representing the area of the base by one third of the altitude.*

IV. *The volume of a sphere equals the product of the number representing the area of the surface by one third of the radius.*

5. How many cubic inches are there in a solid 10 in. long, 8 in. wide, and 10 in. thick?

6. How many cubic feet of air are there in a room 20 ft. long, 15 ft. wide, and 10 ft. high?

7. How many cords of wood are there in a pile 32 ft. long, 4 ft. wide, and 4 ft. high?

8. How many cubic inches are there in a cylinder 10 in. in diameter and 10 in. high?

9. Construct a square prism of soft wood or of a turnip. Ascertain its weight. By cutting the edges with a knife, transform the prism into a pyramid having the same base and the same height. Ascertain its weight. Compare the weight of the pyramid with that of the prism.

10. A prism of a certain size weighs 10 oz. What will be the weight of the largest pyramid that can be constructed from it? State the rule for finding the volume of a pyramid or of a cone.

11. A square prism is 12 in. wide and 30 in. high. What is its volume?

12. A cone is 10 in. in diameter and 10 in. high. Find its volume.

13. How many $\frac{1}{4}$-in. cubes can be cut from a 1-in. cube?

14. How many 2-in. cubes can be cut from a 4-in. cube?

Written

15. How many cubic inches are there in an 8-in. cube? in a 16-in. cube?

16. How many cubic feet of water are there in a tank 16 ft. long, 4 ft. 6 in. wide, and 18 in. deep?

17. What is the cost of excavating a cellar 60 ft. long, 30 ft. wide, and 6 ft. deep, at 10¢ per cubic yard?

Solution. $\frac{60 \times 30 \times 6}{27} = 400$, number of cubic yards.

$400 \times 10¢ = \$40$. *Ans.*

18. Find the cost of excavating for the basement of a store 150 ft. long, 80 ft. wide, and 9 ft. deep, at $8\frac{1}{3}$ cents per cubic yard.

19. How many cubic yards of soil are removed in digging a sewer on a level street 2 mi. long, the ditch being 2 ft. wide and 6 ft. deep?

20. How many cubic yards of soil are removed in digging a ditch for the foundation of a house 64 ft. $\times$ 30 ft., the foundation being 2 ft. 6 in. thick, and extending 2 ft. 6 in. beneath the surface of the ground?

Illustrate the surface of the ditch by a drawing.

21. How many barrels of water will a cistern 11 ft. long, 9 ft. wide, and 7 ft. deep hold?

SOLUTION. $\frac{11 \times 9 \times 7 \times 1728 \times \frac{4}{31}}{231} = 164\frac{4}{31}$, number of barrels.

State the reason for the employment of the different factors in the operation.

22. How many barrels of water are there in a reservoir 44 ft. long, 36 ft. wide, and 14 ft. deep?

23. How many barrels of water are there in a rectangular tank 21 ft. long, 3 ft. 6 in. wide, and 1 ft. 6 in. deep?

24. How many bushels of grain are there in a bin 12 ft. long, 10 ft. wide, and 8 ft. deep?

25. How many cubic inches are there in a cylindrical vessel 8 in. in diameter and 12 in. high?

26. What is the volume of the largest cone that can be constructed from a cylindrical piece of wood 12 in. in diameter and 21 in. high? the volume of the largest sphere?

27. What is the weight of water in a cylindrical bucket 14 in. in diameter and 18 in. high? (A cubic foot of water weighs $62\frac{1}{2}$ lb.)

28. What is the weight of an iron sphere 8 in. in diameter, the iron being $7\frac{1}{8}$ times as heavy as an equal bulk of water?

29. How many cubic inches of water will a cone-shaped funnel 8 in. in diameter and 12 in. high, hold?

30. A circular tank 5 ft. in diameter and $3\frac{1}{2}$ ft. high is filled with water. How many gallons does it contain?

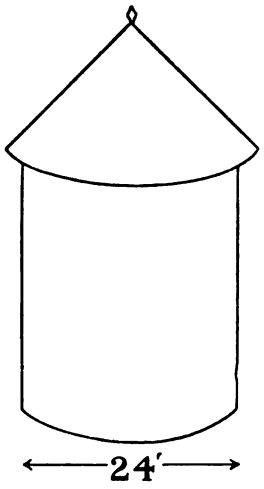
31. What is the capacity in gallons of a tank 6 ft. $\times$ 5 ft. $\times$ 10 ft.?

32. Find the weight of the water when the tank in Ex. 31 is full.

33. A tank to contain 1472.625 gal. is to be 10′ high. What should be its diameter?

34. A man wishes to build a circular cesspool 9′ in diameter and 24′ deep. How many loads of dirt will have to be excavated?

(1 load = 1 cu. yd.)



35. A silo 24′ in diameter is to be constructed so as to contain 250 tons of silage. If silage averages 40 lb. per cubic foot of volume, what must be the depth of the silo?

36. How many bricks laid edgewise would it require to lay the floor of this silo if 2% is allowed for waste in cutting bricks?

37. A silo is built 36′ high and contains 280 tons of silage. What is the diameter of the silo?

REVIEW

PROBLEMS

Oral

1. What is the unit used by contractors in estimating cost of plastering? of papering? of painting? of macadamizing roads? What is the area of the cover of this book?

2. What measure is used in stating the area of a lot? the area of a city block? the area of a farm? by the geographer in stating the area of the country? of the state?

3. What is the area of the floor of a room 12 ft. long and 10 ft. wide? What is the area of a blackboard 16 ft. long and 4 ft. wide? What is the unit of measurement?

4. What is the cost of painting a solid board fence 150 ft. long and 10 ft. high at a cost of 50¢ per square of one hundred square feet? Work in two ways. Which way is the better?

5. A room is 15 ft. long, 12 ft. wide, and 9 ft. high. How many square yards are there in the walls? in the ceiling?

Written

6. What is the cost of painting a floor 18 ft. long and 16 ft. wide at 10¢ per square yard?

7. How many square yards of plastering are there in a hall 90 ft. long, 30 ft. wide, and 20 feet high, no allowance being made for doors, windows, and baseboard?

Solution

$$90 \text{ ft.} \times 2 + 30 \text{ ft.} \times 2 = 240 \text{ ft., length of walls.}$$

$$\frac{240 \times 20}{9} = 533\tfrac{1}{3}, \text{ number square yards in walls.}$$

$$\frac{90 \times 30}{9} = 300, \text{ number square yards in ceiling.}$$

$$533\tfrac{1}{3} + 300 = 833\tfrac{1}{3}, \text{ number square yards of plaster in hall.}$$

8. What is the cost of lathing and plastering a store 120 ft. long, 50 ft. wide, and 22 ft. high, at $33\frac{1}{3}$ cents per square yard, an allowance of 10 sq. yd. being made for doors, windows, and baseboard?

9. What is the cost of plastering a room 18 ft. 6 in. long, 16 ft. 3 in. wide, and 10 ft. 6 in. high, at 15¢ per square yard, allowance being made for one half of the space occupied by three doors and two windows, each 6 ft. 9 in. in height and 3 ft. 6 in. in width? (No allowance for baseboard.)

10. A roll of paper is usually 8 yd. long and $\frac{1}{2}$ yd. wide. What will be the cost of paper for the walls of a room 18 ft. 6 in. long, 16 ft. 3 in. wide, and 10 ft. 6 in. high at 20¢ per roll? (See p. 216.)

11. A dining room in a hotel 50 ft. long, 28 ft. wide, and 19 ft. high is being renovated, — the floor oiled and polished at 5¢ per square foot, the ceiling papered with paper at 50¢ a roll including labor, the walls frescoed at $37\frac{1}{2}$¢ per square yard. Write a receipted bill for the contractor.

12. A room is 18 ft. long and 16 ft. wide. What is the cost of carpet for it at 75¢ a yard, the strips to run crosswise and the carpet being a yard wide? the strips to run lengthwise?

NOTE. In laying the carpet lengthwise, 5 strips of the carpeting 1 yd. wide would cover only 15 ft., leaving a strip of the floor 1 ft. wide and 18 ft. long still to be covered. Hence another strip must be bought. Therefore 6 strips 18 ft. long, or 108 ft., or 36 yd., are needed.

13. At $1.25 a yard, what is the cost of Brussels carpet 27 in. wide for a room 16 ft. 6 in. long and 15 ft. wide, strips to run lengthwise, making an allowance of 9 inches per strip after the first strip is laid for matching designs? What is the cost if the strips run crosswise?

14. The floor of a hall 120 ft. long and 50 ft. wide is being covered with 30-in. carpet, at $1.60 per yard. If 9 in. are allowed on each strip but the first for matching designs, what is the cost of the carpet?

15. What is the value of a piece of land 120 rd. long and 40 rd. wide, at $55 an acre?

16. A map is drawn to a scale of $\frac{1}{4}$ inch to the mile. How many acres are represented by 1 sq. ft. of the map?

17. A road is ordinarily 4 rd. wide. How many miles of it will make 20 A.?

18. What is the cost of bricks for paving a street $2\frac{1}{2}$ mi. long and 4 rd. wide, at $15 per thousand, the bricks being $8\frac{1}{4}$ in. long, 4 in. wide, and $2\frac{1}{4}$ in. thick?

NOTE. The bricks are laid on edge. See p. 248, note.

19. Which is cheaper, and how much, — to macadamize a yard 125 ft. long and 80 ft. wide, at 50¢ a square yard, or to pave it with cut stones 10 in. long and 8 in. wide, at $20 per M?

20. The length of the base of a parallelogram is 60 ft.; its altitude is 39 ft. Find its area.

21. How many acres are there in a farm in the form of a parallelogram 68 rd. long and 45 rd. wide?

22. What is the area of a trapezoid whose parallel sides are 68 rd. and 42 rd., the perpendicular distance between them being $37\frac{1}{2}$ rd.?

23. The area of a trapezoid is 300 sq. in. The perpendicular distance between the two bases is 12 in. If the length of one of the parallel sides is 30 in., what is the length of the other parallel side?

24. What is the area of a triangular piece of land with a base of 62 rd. and a perpendicular distance of 95 rd. from the vertex to the base?

25. What is the cost of a triangular piece of land with a base of 2112 ft. and an altitude of 1650 ft., at $50 per acre?

26. How much cloth is wasted in cutting the largest circle possible from a piece of cloth 16 in. square?

27. What is the difference in area between an 8-in. circle and a 16-in. circle?

28. Find the total atmospheric pressure, at 15 lb. per square inch, upon the top of a circular glass plate 32 in. in diameter.

29. Two circles, 5 in. and 12 in., respectively, in diameter, have the same center. What is the area of the ring between their circumferences?

30. The area of a circle is 1 sq. in. What is its diameter?

31. Find the entire surface of a 6-in. cube; of a 12-in. cube.

32. A crib is 9 ft. 6 in. by 9 ft. 4 in. by 8 ft. 7 in. Find its contents in cubic feet.

33. A wagon box is 10 ft. long, $3\frac{1}{2}$ ft. wide, and 44 in. deep. How many bushels ($1\frac{1}{4}$ cu. ft. each) of wheat will it hold?

34. If the "spaces or middles" between corn rows are $3\frac{1}{2}$ ft. wide, how many rows are there in a field 42 rd. wide? 56 rd. wide?

35. If each shock of corn fodder is 12 hills square ($3\frac{1}{2}$ ft. between hills), how many shocks can be cut from a field 56 rd. wide and 84 rd. long? How many hills are there in the field?

36. A field of oats containing 36.5 A. yields on an average 37.2 bu. per acre. The owner of the land gets .6 and the tenant .4 of the total crop, which is marketed at $.45 per bushel. How much money does each realize from the crop?

37. How many cubic inches of water can be poured into a cone 10 in. in diameter and 15 in. high?

38. A heap of wheat in the corner of a bin is 5 ft. high, and extends 5 ft. each side from the corner. Find the number of bushels in the heap.

39. A 12-inch sphere is reduced to 10 in. How many cubic inches are there in the shell removed?

40. How many square inches of cloth will cover the top and sides of a cabinet 14 in. long, 8 in. wide, and 7 in. high?

41. How many square yards of cement plastering are required for the sides and the bottom of a cistern 10 ft. 3 in. long, 7 ft. 6 in. wide, and 6 ft. deep?

42. What is the lateral surface of a prism 16 in. high, having a triangular base with sides of 4 in., 5 in., and 6 in.?

43. What is the lateral surface of a cylinder 10 in. in diameter and 15 in. high? its entire surface?

44. How many square yards of cement plastering are required for a cylindrical cistern 8 ft. in diameter and 8 ft. deep? (The wall and the bottom of the cistern must be considered.)

45. What is the lateral surface of a triangular prism having a height of 12 in., and having for its base an equilateral triangle the entire length of whose sides is 36 in.?

46. How many square inches of metal are required for tinning the entire surface of a cone having a base 12 in. in diameter and a slant height of 21 in.?

47. What is the area of the surface of a 1-in. sphere? of a 10-in. sphere?

48. At $1.50 per square yard, what is the cost of gilding a 3-ft. globe?

Problems without Numbers

In answering the following questions, make and solve an original problem to show the correctness of your answer. When possible, draw a figure to illustrate your problem.

1. Having given the side of a square, how can you find its area?

2. Having given the area of a square, how can you find its side?

3. Having given the dimensions of a rectangle, how can you find its area?

4. Having given the area of a rectangle and one side, how can you find the other side?

5. Having given the area of a parallelogram and its altitude, how can you find its length?

6. Having given the base and the altitude of a triangle, how can you find its area?

7. Having given the area of a triangle and its base, how can you find its altitude?

8. Having given the hypotenuse and one of the other sides of a right-angled triangle, how can you find the third side?

9. Having given the base and the perpendicular of a right-angled triangle, how can you find the hypotenuse?

10. Having given the sides of an equilateral triangle, how can you find its area?

11. Having given the base and the altitude of an isosceles triangle, how can you find its area?

12. Having given the three sides of any triangle, how can you find its area?

13. Having given the diameter of a circle, how can you find its circumference?

14. Having given the radius of a circle, how can you find its circumference?

15. Having given the circumference of a circle, how can you find its diameter? its radius?

16. Having given the radius of a circle, how can you find its area? Give formula.

17. Having given the area of a circle, how can you find its diameter?

18. Having given the dimensions of a prism, how can you find its lateral surface? its entire surface?

19. Having given the diameter and the altitude of a cylinder, how can you find its lateral surface? its entire surface?

20. What must be known to find the lateral surface of a pyramid or a cone? How can you find it?

21. Having given the diameter of a sphere, how can you find its surface?

22. How can you find the volume of a prism? of a cylinder? of a cone? of a pyramid? of a sphere?

GENERAL REVIEW

Oral

15 14 12 21 33 45 54 76 67 98 89 90 86 75

1. Rapidly add 5 to each of the above numbers; then 6; 4; 2; 3; 8; 7; 9; 10; 12; 11.

2. Rapidly subtract 3 from each of the above numbers; then 2; then 6; 7; 4; 5; 8; 9; 12; 10; 11.

3. Rapidly multiply each of the above numbers by 3; by 4; by 5; by 7; by 6; by 8; by 9; by 10; by 11; by 12.

4. Rapidly divide each of the above numbers by 2; by 4; by 5; by 3; by 6; by 8; by 7; by 9; by 10; by 11; by 12. Give quotients and remainders.

Rapidly make change from $1 for the following amounts:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
5.	56¢	75¢	19¢	38¢	97¢	40¢
6.	62¢	83¢	24¢	26¢	29¢	70¢

Reduce to the lower denomination mentioned:

	a.	*b.*	*c.*	*d.*
7.	2 qt. 1 pt.	1 lb. 2 oz.	2 min. 15 sec.	1 yd. 2 ft.
8.	2 gal. 1 qt.	2 T. 1 lb.	2 hr. 5 min.	1 bu. 1 qt.
9.	3 pk. 1 qt.	3 ft. 2 in.	2 wk. 3 da.	3 lb. 5 oz.
10.	2 bu. 1 pk.	2 yd. 1 in.	1 sq. yd. 1 sq. ft.	2 yd. 5 in.

Reduce to the next higher denomination :

	a.	*b.*	*c.*	*d.*
11.	16 qt. (liq.)	24 in.	18 sq. ft.	120 min.
12.	16 qt. (dry)	9 ft.	48 oz.	135 da.
13.	4 pt.	640 rd.	120 sec.	148 hr.
14.	16 pk.	72 in.	4000 lb.	200 yr.

15. Change $\frac{1}{2}$ to fourths ; to sixths ; to eighths ; to tenths ; to twelfths ; to sixteenths.

16. Change $\frac{1}{3}$ to sixths ; to ninths ; to fifteenths ; to eighteenths.

17. Change $\frac{3}{4}$ to eighths ; to twelfths ; to sixteenths.

18. Change $\frac{4}{5}$ to tenths ; to fifteenths ; to twentieths ; to twenty-fifths.

19. Change $\frac{1}{8}$ to sixteenths ; $\frac{5}{8}$ to sixteenths ; $\frac{7}{8}$ to sixteenths.

20. Change to twelfths : $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{4}{4}$; $\frac{5}{6}$.

21. Change to sixteenths : $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{3}{4}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$.

22. Change $\frac{4}{8}$ to halves ; $\frac{6}{9}$ to thirds ; $\frac{8}{12}$ to thirds ; $\frac{6}{12}$ to sixths.

23. Change to lower terms : $\frac{2}{4}$; $\frac{4}{6}$; $\frac{6}{8}$; $\frac{6}{10}$; $\frac{8}{12}$; $\frac{10}{16}$.

Change to lowest terms :

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
24.	$\frac{2}{4}$	$\frac{3}{9}$	$\frac{6}{8}$	$\frac{10}{15}$	$\frac{15}{20}$	$\frac{12}{15}$	$\frac{10}{16}$	$\frac{10}{12}$
25.	$\frac{4}{6}$	$\frac{4}{10}$	$\frac{8}{12}$	$\frac{12}{16}$	$\frac{15}{25}$	$\frac{10}{20}$	$\frac{9}{12}$	$\frac{4}{20}$

Change to improper fractions :

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
26.	$3\frac{1}{2}$	$6\frac{1}{4}$	$9\frac{2}{5}$	$6\frac{3}{8}$	$3\frac{3}{10}$	$2\frac{3}{16}$
27.	$4\frac{1}{3}$	$7\frac{3}{4}$	$7\frac{1}{6}$	$4\frac{5}{8}$	$4\frac{5}{12}$	$2\frac{1}{20}$
28.	$5\frac{2}{3}$	$8\frac{1}{5}$	$7\frac{5}{6}$	$5\frac{7}{8}$	$1\frac{5}{16}$	$3\frac{1}{25}$

Change to whole or mixed numbers:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
29.	$\frac{4}{2}$	$\frac{7}{3}$	$\frac{10}{5}$	$\frac{15}{8}$	$\frac{24}{8}$	$\frac{22}{11}$	$\frac{28}{14}$	$\frac{64}{16}$
30.	$\frac{5}{2}$	$\frac{12}{4}$	$\frac{14}{5}$	$\frac{18}{9}$	$\frac{35}{8}$	$\frac{36}{12}$	$\frac{30}{16}$	$\frac{50}{25}$
31.	$\frac{6}{3}$	$\frac{15}{4}$	$\frac{12}{6}$	$\frac{25}{10}$	$\frac{40}{10}$	$\frac{39}{12}$	$\frac{49}{16}$	$\frac{40}{20}$

Reduce to fractions having the l. c. d., and add:

	a.	*b.*	*c.*	*d.*
32.	$\frac{1}{2}, \frac{1}{4}$	$\frac{1}{2}, \frac{3}{4}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$
33.	$\frac{1}{2}, \frac{1}{3}$	$\frac{1}{4}, \frac{3}{8}$	$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$	$\frac{1}{4}, \frac{1}{5}, \frac{1}{10}$
34.	$\frac{1}{3}, \frac{1}{5}$	$\frac{1}{8}, \frac{3}{16}$	$\frac{1}{2}, \frac{1}{8}, \frac{1}{16}$	$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

Subtract:

	a.	*b.*	*c.*	*d.*	*e.*
35.	$\frac{1}{2} - \frac{1}{4}$	$\frac{3}{4} - \frac{3}{8}$	$\frac{1}{2} - \frac{5}{12}$	$\frac{3}{5} - \frac{3}{10}$	$\frac{3}{4} - \frac{2}{5}$
36.	$\frac{1}{2} - \frac{1}{8}$	$\frac{5}{6} - \frac{1}{2}$	$\frac{5}{8} - \frac{3}{16}$	$\frac{3}{8} - \frac{5}{16}$	$\frac{5}{8} - \frac{1}{3}$
37.	$\frac{1}{2} - \frac{1}{16}$	$\frac{7}{8} - \frac{3}{4}$	$\frac{7}{12} - \frac{1}{6}$	$\frac{2}{3} - \frac{1}{6}$	$\frac{2}{3} - \frac{5}{12}$

Find products:

	a.	*b.*	*c.*	*d.*	*e.*
38.	$\frac{2}{3}$ of 36	$4 \times \frac{3}{8}$	$\frac{2}{5}$ of $\frac{5}{8}$	$\frac{5}{8}$ of $\frac{3}{10}$	$\frac{5}{12}$ of $\frac{3}{10}$
39.	$\frac{3}{8}$ of 48	$5 \times \frac{7}{15}$	$\frac{3}{4}$ of $\frac{4}{5}$	$\frac{2}{3}$ of $\frac{3}{7}$	$\frac{3}{16}$ of $\frac{8}{9}$
40.	$\frac{5}{16}$ of 64	$4 \times 2\frac{1}{4}$	$\frac{5}{16}$ of $\frac{4}{5}$	$\frac{2}{3}$ of $\frac{3}{16}$	$\frac{3}{10}$ of $\frac{5}{6}$

Find quotients:

	a.	*b.*	*c.*	*d.*	*e.*
41.	$\frac{3}{4} \div 3$	$3 \div \frac{3}{4}$	$\frac{3}{4} \div \frac{3}{8}$	$\frac{3}{8} \div \frac{2}{3}$	$\frac{5}{6} \div \frac{5}{12}$
42.	$\frac{4}{5} \div 4$	$4 \div \frac{4}{5}$	$\frac{3}{8} \div \frac{3}{4}$	$\frac{3}{10} \div \frac{3}{5}$	$\frac{5}{12} \div \frac{5}{6}$
43.	$\frac{4}{5} \div 5$	$5 \div \frac{4}{5}$	$\frac{2}{3} \div \frac{3}{8}$	$\frac{3}{5} \div \frac{3}{10}$	$\frac{5}{16} \div \frac{5}{8}$

Find what part the first number is of the second:

	a.	*b.*	*c.*	*d.*	*e.*
44.	3, 4	10, 6	8, 6	9, 6	$\frac{1}{4}$, $\frac{1}{2}$
45.	4, 5	9, 36	6, 8	18, 4	$\frac{3}{8}$, $\frac{3}{4}$
46.	6, 10	36, 9	16, 3	32, 24	$\frac{3}{16}$, $\frac{3}{8}$

Find the ratio of:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
47.	3 to 4	4 to 20	6 to 8	10 to 15	$\frac{1}{4}$ to $\frac{1}{8}$	$\frac{3}{4}$ to $\frac{3}{16}$
48.	4 to 3	20 to 4	8 to 6	15 to 10	$\frac{1}{8}$ to $\frac{1}{4}$	$\frac{3}{16}$ to $\frac{3}{4}$

Find the number of which:

	a.	*b.*	*c.*	*d.*	*e.*
49.	4 is $\frac{1}{2}$	8 is $\frac{4}{5}$	12 is $\frac{4}{5}$	20 is $\frac{5}{6}$	18 is $\frac{3}{8}$
50.	6 is $\frac{1}{3}$	9 is $\frac{3}{5}$	14 is $\frac{7}{8}$	25 is $\frac{5}{8}$	16 is $\frac{4}{5}$
51.	7 is $\frac{1}{5}$	10 is $\frac{5}{8}$	16 is $\frac{2}{3}$	30 is $\frac{3}{10}$	24 is $\frac{3}{8}$

Reduce to decimals and to per cents:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*j.*
52.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{16}$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{12}$	$\frac{1}{25}$
53.	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{1}{20}$

Reduce to common fractions in their lowest terms:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*
54.	.50	.40	.12$\frac{1}{2}$	.62$\frac{1}{2}$	.16$\frac{2}{3}$	.66$\frac{2}{3}$	.06$\frac{1}{4}$
55.	.25	.75	.37$\frac{1}{2}$	.87$\frac{1}{2}$	.33$\frac{1}{3}$	.83$\frac{1}{3}$	.04

Find the cost of 48 articles at the following prices:

	a.	*b.*	*c.*	*d.*	*e.*
56.	10¢	50¢	12$\frac{1}{2}$¢	33$\frac{1}{3}$¢	$1.25
57.	20¢	60¢	37$\frac{1}{2}$¢	66$\frac{2}{3}$¢	$1.75
58.	25¢	75¢	62$\frac{1}{2}$¢	16$\frac{2}{3}$¢	1.37\frac{1}{2}$
59.	50¢	80¢	87$\frac{1}{2}$¢	83$\frac{1}{3}$¢	1.33\frac{1}{3}$

How many articles can be purchased for \$10 at each of the following prices:

	a.	*b.*	*c.*	*d.*	*e.*
60.	10¢	25¢	$16\frac{2}{3}$¢	$62\frac{1}{2}$¢	$87\frac{1}{2}$¢
61.	20¢	$12\frac{1}{2}$¢	$33\frac{1}{3}$¢	$83\frac{1}{3}$¢	$6\frac{1}{4}$¢

Find:

	10 % of:	50 % of:	75 % of:	$12\frac{1}{2}$ % of:	$33\frac{1}{3}$ % of:
62.	10	12	4	8	10
63.	20	16	8	16	20
64.	50	18	12	24	30

Find what per cent the first number is of the second:

	a.	*b.*	*c.*	*d.*	*e.*
65.	5 of 10	3 of 9	4 of 16	5 of 8	2 of 3
66.	2 of 20	4 of 12	3 of 4	8 of 5	3 of 2

Of what number is

	a.	*b.*	*c.*	*d.*
67.	2, 50 %	6, 25 %	8, 10 %	15, $37\frac{1}{2}$ %
68.	3, $33\frac{1}{3}$ %	7, 20 %	9, $12\frac{1}{2}$ %	16, 25 %

Find the profit or loss:

	a.		*b.*		*c.*	
	Cost	Rate of Gain	Cost	Rate of Loss	Cost	Rate of Gain
69.	\$10	10 %	\$4	25 %	\$ 9	$33\frac{1}{3}$ %
70.	5	20 %	8	$12\frac{1}{2}$ %	10	20 %

Find the per cent of profit or loss:

	a.		*b.*		*c.*	
	Cost	Selling Price	Cost	Selling Price	Cost	Selling Price
71.	\$.10	\$.15	\$10	\$18	\$10	\$8
72.	\$.10	\$.12	\$ 9	\$12	\$10	\$9

Find the discount and the net proceeds:

	a.		*b.*		*c.*	
	List Price	Rate of Disc.	List Price	Rate of Disc.	List Price	Rates of Disc.
73.	$10	10 %	$36	33⅓ %	$10	20 %, 10 %
74.	$40	25 %	$20	20 %	$40	25 %, 20 %

Find the commission:

	Sales	Rate of Com.	Sales	Rate of Com.	Sales	Rate of Com.
75.	$24	25 %	$18	33⅓ %	$10	10 %
76.	$40	20 %	$16	12½ %	$20	25 %

Find the premium on the following insurance policies:

	Face of Policy	Rate	Face of Policy	Rate	Face of Policy	Rate
77.	$1000	25 ¢ per 100	$1000	⅕ %	$8000	⅛ %
78.	$1500	20 ¢ per 100	$2000	⅖ %	$4000	⅜ %

Find the taxes on the following pieces of property:

	Assessment	Tax Rate	Assessment	Tax Rate	Assessment	Tax Rate
79.	$1000	.02	$1600	.05	$3000	.01½
80.	$1200	.04	$2000	.01¼	$3200	.02¼

Find the duty:

	Value	Ad Val. Duty	Value	Ad Val. Duty	Quantity	Specific Duty
81.	$100	10 %	$3000	33⅓ %	200 lb.	$1 a lb.
82.	$200	20 %	$4000	25 %	100 bu.	5 ¢ a bu.

Find the interest and the amount of:

83. $100, 5 yr., 5 %
84. $200, 6 yr., 3 %
85. $1000, 2 yr., 2½ %
86. $300, 2½ yr., 4 %
87. $600, 3⅓ yr., 6 %
88. $1000, 4 yr., 3½ %

Written

Time yourself with these exercises. Then work some of them again, trying to beat your record.

Write from dictation, add, and test results:

	a.	*b.*	*c.*	*d.*
89.	653127	805672	$9324.73	$7894.67
	894238	790745	6849.89	5639.86
	757899	983286	7787.75	8427.95
	562328	648354	3538.79	6458.43
	429899	573729	4305.84	7894.09
	701572	784648	5664.68	8345.05
	834999	896349	8976.88	7263.97

90.	.0356	18.09	256.0407	785.05
	12.7043	5.0425	14.953	25
	13.02	184.7653	2157.647	185.0003
	15.6351	3157.634	3082.0057	.0007
	17.7401	463.05	9743.0562	7.6534

	a.	*b.*	*c.*	*d.*	*e.*
91.	$12\frac{3}{4}$	$13\frac{3}{5}$	$125\frac{2}{3}$	$62\frac{1}{2}$	$75\frac{3}{8}$
	$156\frac{7}{8}$	$14\frac{3}{10}$	$356\frac{5}{9}$	$37\frac{3}{4}$	$87\frac{3}{10}$
	$54\frac{1}{2}$	$17\frac{2}{15}$	$714\frac{5}{18}$	$16\frac{2}{3}$	$15\frac{3}{16}$

	a.	*b.*	*c.*
92.	13 ft. 4 in.	14 pk. 4 qt.	16 lb. 5 oz.
	15 ft. 10 in.	15 pk. 5 qt.	23 lb. 14 oz.
93.	4 gal. 3 qt.	3 yd. 2 ft.	10 mi. 210 rd.
	3 gal. 2 qt.	4 yd. 1 ft.	50 mi. 240 rd.

Subtract and test:

	a.	*b.*	*c.*	*d.*
94.	563412	721000	$8205.72	$4000.00
	486789	356879	4056.98	3678.29
95.	213.05	356.0057	562.	1825.095
	12.9746	64.92	.0576	746.9
96.	$112\frac{3}{4}$	$113\frac{1}{6}$	$169\frac{5}{16}$	$562\frac{5}{12}$
	$15\frac{7}{8}$	$17\frac{2}{3}$	$143\frac{3}{8}$	$314\frac{5}{16}$

	a.	*b.*	*c.*
97.	15 ft. 8 in.	25 lb. 8 oz.	25 wk. 4 da.
	13 ft. 10 in.	16 lb. 10 oz.	16 wk. 6 da.

Find the difference in time between:

98. Aug. 15, 1913, and Jan 4, 1915. April 20, 1913, and Feb. 25, 1915.

99. Sept. 22, 1912, and Feb. 16, 1914. Sept. 29, 1912, and Mar. 12, 1914.

Make change from $1 and $2 for each of the purchases in column *a*; from $5 and $10 for each of the purchases in column *b*; from $15 and $20 for each of the purchases in columns *c*, *d*, and *e*.

	a.	*b.*	*c.*	*d.*	*e.*
100.	$.78	$1.54	$12.36	$14.98	$10.75
101.	$.65	$2.75	$10.16	$13.52	$11.98
102.	$.39	$3.65	$10.54	$12.39	$12.16
103.	$.97	$4.94	$11.25	$14.83	$14.45

Find the products, canceling when possible. Test answers:

	a.	*b.*	*c.*
104.	3214×3256	$\frac{5}{6} \times \frac{3}{4}$	6.75×1.02375
105.	1056×2097	$\frac{7}{8} \times \frac{4}{5}$	$.072 \times .072$
106.	9457×9457	$13\frac{3}{16} \times 14\frac{3}{8}$	1.052×6.04
107.	$2360 \times \$28.97$	$15\frac{3}{5} \times 16\frac{5}{8}$	70.3×60.005
108.	$2745 \times \$102.965$	$205\frac{5}{16} \times 13\frac{3}{4}$	$.054 \times \$17.65$
109.	6234×1.002	$30\frac{2}{15} \times 14\frac{3}{10}$	$.065 \times \$118.54$
110.	75.15×91.005	$54\frac{3}{20} \times 12\frac{3}{10}$	$.037\frac{1}{2} \times 200$
111.	813.237×16.095	$15\frac{5}{16} \times 19\frac{3}{8}$	$.062\frac{1}{2} \times 800$

Find the quotients and remainders and test the answers:

112.	$59492 \div 573$	$\frac{5}{6} \div \frac{2}{3}$	3 ft. 6 in. ÷ 6
113.	$120650 \div 457$	$\frac{7}{8} \div \frac{3}{4}$	4 gal. 2 qt. ÷ 9
114.	$\$5942.97 \div 621$	$15\frac{3}{4} \div 5\frac{1}{4}$	5 lb. 1 oz. ÷ 9
115.	$78105 \div 635$	$5\frac{3}{10} \div 4\frac{3}{5}$	7 wk. 3 da. ÷ 12
116.	$5252.88 \div 1.527$	$60\frac{1}{8} \div 15\frac{3}{4}$	5 doz. 4 ÷ 8
117.	$37366.692 \div 12.046$	$12\frac{4}{5} \div 1\frac{3}{8}$	5 yr. 4 mo. ÷ 16
118.	$5372506 \div 15.007$	$10\frac{1}{8} \div 2\frac{1}{4}$	4 pk. 4 qt. ÷ 12
119.	$318.0848 \div 1.5004$	$7\frac{1}{5} \div 1\frac{1}{6}$	4 yd. 3 ft. ÷ 5

Find the ratio of:

120.	15 to 60	$\frac{3}{4}$ to $\frac{4}{5}$	2 in. to 3 ft.
121.	60 to 15	$\frac{4}{5}$ to $\frac{3}{4}$	2 gal. to 3 qt.
122.	$12\frac{1}{2}$ to $37\frac{1}{2}$	$\frac{3}{4}$ to $\frac{7}{8}$	2 oz. to 1 lb.
123.	$37\frac{1}{2}$ to $12\frac{1}{2}$	$\frac{7}{8}$ to $\frac{3}{4}$	1 lb. to 2 oz.

Reduce to improper fractions:

	a.	*b.*	*c.*	*d.*
124.	$3\frac{1}{2}$	$14\frac{3}{4}$	$15\frac{3}{8}$	$83\frac{1}{3}$
125.	$4\frac{2}{3}$	$15\frac{5}{8}$	$33\frac{1}{3}$	$20\frac{5}{12}$
126.	$5\frac{3}{4}$	$16\frac{2}{3}$	$62\frac{1}{2}$	$19\frac{3}{5}$
127.	$7\frac{3}{8}$	$25\frac{5}{12}$	$87\frac{1}{2}$	$43\frac{4}{5}$

Reduce to whole or to mixed numbers:

	a.	*b.*	*c.*	*d.*	*e.*
128.	$\frac{50}{3}$	$\frac{49}{4}$	$\frac{56}{2}$	$\frac{43}{5}$	$\frac{31}{6}$
129.	$\frac{49}{7}$	$\frac{28}{8}$	$\frac{81}{9}$	$\frac{45}{10}$	$\frac{65}{12}$
130.	$\frac{78}{15}$	$\frac{64}{16}$	$\frac{100}{20}$	$\frac{75}{25}$	$\frac{35}{11}$

Reduce to lowest terms:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*
131.	$\frac{14}{16}$	$\frac{10}{15}$	$\frac{16}{24}$	$\frac{18}{36}$	$\frac{16}{20}$	$\frac{14}{35}$
132.	$\frac{36}{60}$	$\frac{21}{49}$	$\frac{24}{64}$	$\frac{25}{100}$	$\frac{15}{75}$	$\frac{16}{40}$
133.	$\frac{18}{81}$	$\frac{16}{56}$	$\frac{27}{63}$	$\frac{36}{144}$	$\frac{27}{72}$	$\frac{20}{48}$

Reduce to fractions having the l. c. d.:

	a.	*b.*	*c.*
134.	$\frac{1}{2}, \frac{2}{3}, \frac{1}{4}$	$\frac{2}{3}, \frac{3}{5}, \frac{3}{10}$	$\frac{1}{6}, \frac{2}{5}, \frac{3}{10}$
135.	$\frac{3}{4}, \frac{1}{6}, \frac{2}{5}$	$\frac{3}{8}, \frac{5}{12}, \frac{3}{16}$	$\frac{3}{4}, \frac{5}{12}, \frac{5}{6}$
136.	$\frac{1}{2}, \frac{3}{4}, \frac{3}{8}$	$\frac{1}{2}, \frac{3}{8}, \frac{3}{16}$	$\frac{3}{5}, \frac{3}{4}, \frac{3}{10}$

Reduce to decimals and to per cents:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*
137.	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{7}{12}$	$\frac{3}{25}$
138.	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{3}{5}$	$\frac{1}{7}$	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{7}{50}$
139.	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{5}{12}$	$\frac{11}{12}$	$\frac{17}{20}$
140.	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{11}{25}$

Reduce to per cents and to common fractions in their lowest terms:

	a.	*b.*	*c.*	*d.*	*e.*
141.	.20	.12½	.16⅔	1.75	1.12½
142.	.40	.37½	.33⅓	1.25	1.33⅓
143.	.50	.62½	.66⅔	1.50	1.37½
144.	.60	.87½	.83⅓	1.40	1.87½
145.	.25	.06¼	.08⅓	1.60	1.66⅔

Reduce the numbers in column (1) to the denominations indicated in column (2):

	a.		*b.*		*c.*	
	(1) Number	(2) Denomination	(1) Number	(2) Denomination	(1) Number	(2) Denomination
146.	16 pt.	quarts	5 lb.	ounces	5 sq. yd.	sq. feet
147.	5 qt.	pints	80 oz.	pounds	27 sq. ft.	sq. yards
148.	4 gal.	quarts	4000 lb.	tons	3 A.	sq. rods
149.	24 qt.	gal.	3 T.	pounds	54 cu. ft.	cu. yards
150.	40 qt.	pecks	36 in.	feet	5 cu. yd.	cu. feet
151.	5 pk.	quarts	4 ft.	inches	120 sec.	minutes
152.	100 pk.	bushels	72 in.	yards	5 hr.	minutes
153.	16 bu.	pecks	18 ft.	yards	180 min.	hours

Find the cost of:

	a.	*b.*	*c.*
154.	20 lb. @ 25¢	48 yd. @ 83⅓¢	50 yd. @ $1.50
155.	26 lb. @ 50¢	48 yd. @ 37½¢	25 lb. @ $1.25
156.	28 lb. @ 75¢	36 yd. @ 16⅔¢	28 yd. @ $1.40
157.	50 lb. @ 20¢	72 yd. @ 87½¢	50 bu. @ $1.75
158.	60 lb. @ 40¢	40 yd. @ 62½¢	56 gal. @ $1.62½
159.	24 lb. @ 12½¢	64 yd. @ 6¼¢	64 yd. @ $1.87½
160.	33 lb. @ 33⅓¢	80 yd. @ 18¾¢	96 lb. @ $1.37½
161.	42 lb. @ 66⅔¢	84 yd. @ 8⅓¢	66 lb. @ $1.33⅓

Find in the shortest way the number of yards that can be bought for the given sums at the given prices:

	a.		*b.*	
	Amount	Price per Yard	Amount	Price per Yard
162.	\$10	\$.50	\$36	\$.33⅓
163.	\$40	\$.25	\$72	\$.66⅔
164.	\$36	\$.75	\$85	\$.83⅓
165.	\$88	\$.40	\$96	\$.06¼
166.	\$17	\$.12½	\$84	\$1.20
167.	\$15	\$.37½	\$1.21	\$1.37½
168.	\$25	\$.62½	\$1.30	\$1.62½
169.	\$49	\$.87½	\$1.50	\$1.87½

Find:

	a.	*b.*	*c.*	*d.*	*e.*
	20 % of:	12½ % of:	133⅓ % of:	87½ % of:	16⅔ % of:
170.	50	88	\$660	104	\$600
171.	75	160	\$720	128	\$780
172.	100	248	\$540	360	\$834
	8⅓ % of:	6¼ % of:	18¾ % of:	83⅓ % of:	66⅔ % of:
173.	120	320	\$800	180	\$330
174.	624	480	\$960	240	\$450
175.	840	640	\$1440	846	\$792

176. Find 6% of each of the numbers in examples 170 to 175; then 5% of them; 4½%; 3½%; 2½%; 7%; 7½%; 18%; 23%; 45%; 67%; 125%; 115%; 150%.

Find what per cent the first number is of the second:

	a.	*b.*	*c.*
177.	15 of 45.	16 of 48.	1 in. of 1 ft.
178.	45 of 15.	48 of 16.	2 oz. of 2 lb.
179.	12½ of 62½.	⅘ of ¾.	4 qt. of 1 pk.
180.	62½ of 12½.	¾ of ⅘.	2 qt. of 2 gal.

Of what number is:

	a.	*b.*	*c.*
181.	15, 3%	\$ 220, 137½%	\$3500, 62½%
182.	16, 4%	\$2500, 40%	\$7210, 87½%
183.	72, 12½%	\$8100, 112½%	\$8100, 75%
184.	66, 66⅔%	\$7200, 120%	\$7500, 125%

Find the profit or loss:

	Cost	Rate of Gain	Cost	Rate of Loss	Cost	Rate of Gain
185.	\$50	20%	\$75	45%	\$64.30	10%
186.	\$33	33⅓%	\$240	24%	\$56.80	12½%
187.	\$44	25%	\$690	18%	\$20.50	20%
188.	\$64.08	12½%	\$544	26%	\$14.40	28%

Find the per cent of profit or loss:

	Cost	Selling Price	Cost	Selling Price	Cost	Selling Price
189.	\$20	\$24	\$90	\$150	\$25.75	\$20.60
190.	\$24	\$20	\$120	\$90	\$40.20	\$50.25
191.	\$56	\$70	\$150	\$165	\$28.32	\$31.86
192.	\$70	\$56	\$165	\$150	\$36.16	\$27.12

Find the discount and the net price of each of the following:

	List Price	Rate of Discount	List Price	Rates of Discount	List Price	Rates of Discount
193.	\$56	25%	\$120	33⅓%, 25%	\$900	33⅓%, 10%
194.	\$80	12½%	\$600	20%, 10%	\$1000	20%, 10%
195.	\$3.30	33⅓%	\$480	16⅔%, 10%	\$5000	50%, 5%
196.	\$4.50	20%	\$800	12½%, 5%	\$9600	37½%, 10%

Find the commission:

	a.		b.		c.	
	Sales	Rate of Com.	Sales	Rate of Com.	Purchase	Rate of Com.
197.	$3600	25%	$1040	18%	$8800	$2\frac{1}{2}$%
198.	$4800	$12\frac{1}{2}$%	$2020	15%	$7500	5%
199.	$5000	20%	$6000	$33\frac{1}{3}$%	$6300	12%
200.	$6000	$16\frac{2}{3}$%	$4040	25%	$7500	18%

Find the premium on each of the following policies:

	a.		b.	
	Face of Policy	Rate	Face of Policy	Rate
201.	$5000	24¢ per $100	$4000	$\frac{1}{4}$%
202.	$6350	64¢ per $100	$16,000	$\frac{3}{4}$%
203.	$10,000	$1.50 per $100	$15,000	$\frac{3}{5}$%
204.	$16,000	$12\frac{1}{2}$¢ per $100	$20,000	$\frac{4}{5}$%

Find the amount of taxes on the following property:

	a.		b.		c.	
	Assessment	Tax Rate	Assessment	Tax Rate	Assessment	Tax Rate
205.	$15,000	$.024	$10,000	$2\frac{3}{4}$%	$9500	$5\frac{1}{4}$ mills
206.	$18,000	$.0015	$50,600	$5\frac{1}{2}$%	$18,000	$4\frac{1}{2}$ mills
207.	$24,256	$.004	$75,250	$1\frac{1}{4}$%	$24,000	$2\frac{1}{8}$ mills

Find the duty on each of the following imports:

	Value	Ad Valorem Duty	Value	Ad Valorem Duty	Quantity	Specific Duty
208.	$500	30%	$12,000	25%	2000 lb.	$1.85 a pound
209.	$650	35%	$15,040	20%	200 bu.	$.06 a bushel
210.	$800	60%	$10,500	15%	500 doz.	$1.20 a dozen
211.	$900	45%	$55,600	40%	1600 tons	$.50 a ton

Find the interest and amount of:

	Principal	Time	Rate
212.	$2000	3 yr. 2 mo.	6%
213.	$430.50	9 mo.	6%
214.	$5650	1 yr. 6 mo.	5%
215.	$670.70	9 mo. 6 da.	$4\frac{1}{2}$%
216.	$70.95	6 mo. 12 da.	6%
217.	$63.42	2 yr. 3 mo. 6 da.	4%
218.	$840.90	3 yr. 6 mo. 6 da.	$3\frac{1}{2}$%
219.	$5320	1 yr. 4 mo. 10 da.	5%
220.	$6250	1 yr. 4 mo. 10 da.	$3\frac{1}{2}$%
221.	$575.80	5 yr. 4 mo. 1 da.	$2\frac{1}{2}$%
222.	$8900	6 mo.	$5\frac{1}{2}$%
223.	$650.40	4 yr. 3 mo. 10 da.	$3\frac{3}{4}$%

Find the date of maturity, the term of discount, the bank discount, and the proceeds of the following notes:

	Face	Time	Date of Note	Date of Discount	Rate of Discount
224.	$2400	6 mo.	Jan. 2	Feb. 2	5%
225.	$3200	3 mo.	Mar. 1	Mar. 15	6%
226.	$5600	4 mo.	Apr. 3	May 3	5%
227.	$7500	2 mo.	July 6	July 15	4%
228.	$5000	90 da.	May 1	May 15	6%
229.	$6000	60 da.	June 1	June 15	5%
230.	$7000	30 da.	Aug. 15	Sept. 1	$5\frac{1}{2}$%
231.	$8500	6 mo.	Sept. 7	Sept. 15	6%
232.	$9500	5 mo.	Oct. 1	Oct. 15	4%
233.	$10,000	4 mo.	Dec. 1	Dec. 15	5%

Find the cost of the following $100 shares of stocks and $1000 bonds, adding $\frac{1}{8}$ % for brokerage:

	a.		*b.*		*c.*		
	No. of Shares	Market Price	No. of Shares	Market Price	No. of Bonds	Market Price	Rate of Int.
234.	100	$88\frac{3}{8}$	5	119	5	$88\frac{3}{8}$	5 %
235.	500	$96\frac{7}{8}$	10	$102\frac{7}{8}$	10	$103\frac{1}{2}$	4 %
236.	50	$26\frac{1}{8}$	20	$124\frac{1}{2}$	20	$96\frac{7}{8}$	$3\frac{1}{2}$ %
237.	10	$100\frac{1}{2}$	100	$35\frac{3}{4}$	100	$102\frac{1}{4}$	4 %

238. Find the amount of interest on the bonds in examples 234 *c* to 237 *c* and the per cent of income.

Square the following numbers:

	a.	*b.*	*c.*	*d.*	*e.*
239.	13	17	22	125	436
240.	14	18	23	100	569
241.	15	19	24	500	718
242.	10	21	25	600	650
243.	16	36	49	106	720

Find the square root of:

	a.	*b.*	*c.*	*d.*
244.	576	324	5184	11,025
245.	529	1024	3025	11,881
246.	441	2025	4489	48,841
247.	361	9801	5329	112,896
248.	484	10,000	7225	60,025
249.	289	8649	7744	132,496

FARM PROBLEMS

Dairying

To THE TEACHER. Problems on pages 455–469 are intended especially for rural schools and may be omitted, if desired.

NOTE. In all these problems, compute per cents to hundredths of a per cent, amounts of butter fat and skimmed milk to hundredths of a pound, prices of a pound to mills, and final costs to the nearest cent.

Many creameries are owned by the patrons who furnish the milk. Officials elected by the patrons supervise the management of the factory, the sale of the butter, etc. At the close of each month a statement is formulated, showing the amount received from the sale of the butter, the expenses, and the balance that is to be divided among the patrons, in proportion to the amount of cream furnished by each.

The totals of a creamery of 10 patrons were as follows for the month of March:

NAME	POUNDS MILK	AVERAGE TEST	POUNDS BUTTER FAT
		Per Cent	
1. John Smith	6450	4.02	259.29
2. Captain Doe	4326	3.35	
3. James Dean	7640	3.39	
4. Henry King	3064	4.05	
5. Peter Rula	5675	4.25	
6. P. G. Riley	6940	4.20	
7. Roy Gimlin	5045	3.85	
8. Fred Johns	2480	3.96	
9. John Lund	4965	4.15	
10. W. Bell	2983	3.83	

1. The butter for the month was sold for $634.85, and the expenses were $75.97. Find the amount due each patron.

SUGGESTION. First find the amount of butter fat furnished by each patron (4.02% = 4.02 lb. per hundred pounds of milk). Divide the net proceeds by the total number of pounds of butter fat. This gives the price of a pound of butter fat. Multiply the number of pounds furnished by the different patrons by the price of one pound.

2. About 80 lb. of skimmed milk is returned for every 100 lb. of milk furnished. This is worth about 20¢ a hundred. What is the total gross value of the milk handled in this creamery?

3. L. M. Carpenter's milk sheet for the month of February showed the following entries:

1. 245 lb.	*8.* 240 lb.	*15.* 209 lb.	*22.* 250 lb.
2. 256 lb.	*9.* 237 lb.	*16.* 242 lb.	*23.* 245 lb.
3. 199 lb.	*10.* 225 lb.	*17.* 219 lb.	*24.* 256 lb.
4. 244 lb.	*11.* 210 lb.	*18.* 229 lb.	*25.* 260 lb.
5. 263 lb.	*12.* 209 lb.	*19.* 224 lb.	*26.* 262 lb.
6. 213 lb.	*13.* 230 lb.	*20.* 218 lb.	*27.* 260 lb.
7. 245 lb.	*14.* 217 lb.	*21.* 232 lb.	*28.* 275 lb.

Three tests were taken during the month, giving these results: 3.95%, 3.99%, and 4.30%. What was the average per cent of butter fat?

4. If the net price of butter fat was 25¢ a pound, find the amount due Mr. Carpenter.

5. What is the total value of his milk industry for the month? (For data for skimmed milk, see Example 2.)

The following figures are based on the records of a part of the herd of an Iowa farmer:

No. of Cow	Pounds of Milk	Av. Test Per Cent	Av. Price of Butter Fat
1	4832	3.35	33.6¢
2	5488	4.06	34.4¢
3	5896	3.98	33.6¢
4	6808	3.91	32.8¢
5	6972	3.67	34.8¢
6	5093	3.63	32.7¢
7	6771	3.39	32.7¢
8	5557	2.93	32.9¢
9	7489	3.89	33.2¢
10	6215	4.44	34.0¢

Find the pounds of butter fat and its value for :

6. Cow No. 1.
7. Cow No. 2.
8. Cow No. 3.
9. Cow No. 4.
10. Cow No. 5.
11. Cow No. 6.
12. Cow No. 7.
13. Cow No. 8.
14. Cow No. 9.
15. Cow No. 10.

16. Find the total number of pounds of milk produced.

17. How many pounds of butter fat did the ten cows produce?

18. What was the total value of butter fat produced?

19. What was the average number of pounds of milk produced?

20. Which cows were above the average in milk production?

21. What was the average per cent of butter fat?

22. Which cows were above the average in per cent of butter fat?

23. Which cows were above the average in both milk production and in per cent of butter fat?

24. What was the average number of pounds of butter fat produced?

The cost of feeding this herd was as follows :

No. of Cow	Cost of Roughage	Cost of Grain
1	$17.43	$15.77
2	$22.43	$17.41
3	$22.43	$17.41
4	$22.43	$17.04
5	$22.43	$17.41
6	$22.43	$13.97
7	$22.43	$14.77
8	$22.43	$13.97
9	$22.43	$14.34
10	$22.43	$17.41

25. Find the total cost of feeding for each cow.

26. What was the profit on each cow?

27. What was the total profit?

28. Find the average profit per cow.

29. Which cows were above the average in profit?

Find the cost per pound of producing butter fat, and the value of butter fat produced for $1 for:

30. Cow No. 1.
31. Cow No. 2.
32. Cow No. 3.
33. Cow No. 4.
34. Cow No. 5.
35. Cow No. 6.
36. Cow No. 7.
37. Cow No. 8.
38. Cow No. 9.
39. Cow No. 10.

40. What was the average cost per pound of producing the butter fat?

41. Which cows were below the average in cost of production?

A farmer who was a member of a Cow Testing Association had the following record on his herd:

No. of Cow	Pounds of Milk	Av. Test Per Cent	Av. Price of Butter Fat
1	3552	3.89	34.6 ¢
2	2109	4.09	29.0 ¢
3	2682	3.84	30.7 ¢
4	5189	3.93	33.4 ¢
5	4414	4.41	32.0 ¢
6	6959	3.96	33.7 ¢
7	2848	3.88	30.8 ¢
8	1846	3.81	31.2 ¢
9	4780	3.81	34.9 ¢
10	5622	4.01	34.1 ¢

During the period of testing, the cost of feed was as follows:

No. of Cow	Cost of Roughage	Cost of Grain
1	$13.67	$16.43
2	$12.50	$10.50
3	$13.67	$10.71
4	$13.67	$10.71
5	$13.67	$10.71
6	$13.67	$10.71
7	$13.67	$10.71
8	$13.67	$10.71
9	$13.67	$19.63
10	$13.67	$19.63

42. Find the number of pounds of butter fat produced by each cow.

43. What was the total number of pounds of butter fat produced by the herd?

44. What was the value per cow of the butter fat produced?

45. What was the total cost of feed per cow?

46. What was the profit per cow?

47. What was the cost per pound of producing the butter fat for each cow?

48. What was the value of butter fat produced for each dollar expended for feed in the case of each cow?

49. Which cows produced more than the average amount of milk? Which produced more than the average amount of butter fat?

The following is a sample daily ration that was used during the testing period:

10 pounds timothy hay
6 pounds corn
5 pounds bran
2 pounds linseed meal

50. At $12 a ton for timothy hay, $.01 a pound for corn, $.012 a pound for bran, and $.017 a pound for linseed meal, find the cost of this ration.

The following figures are based on a study made by the United States Department of Agriculture of 719 cows owned in Southern states. The data cover a period of 12 months.

	Average Cow	Best Cow	Poorest Cow	Av. of Best Ten Cows	Av. of Poorest Ten Cows
Milk, pounds . .	4299.4	8325.5	1125	8181.9	1577.6
Butter fat . . .	216.8	538.8	64.1	459.0	77.2
Value of skim milk	$8.17	$15.57	$2.12	$16.45	$3.00
Cost of feed per cow . . .	$36.27	$72.03	$23.80	$65.73	$24.63

Find the value of butter fat @ $.34 produced by:

51. Average cow.

52. Best cow.

53. Poorest cow.

54. Average of best ten cows.

55. Average of poorest ten cows.

Find the difference in value between butter fat and skim milk produced by:

56. Average cow.

57. Best cow.

58. Poorest cow.

59. Average of best ten cows.

60. Average of poorest ten cows.

Deducting the cost of feed, find the total profit in each case on the butter fat and skim milk produced by:

61. Average cow.

62. Best cow.

63. Poorest cow.

64. Average of best ten cows.

65. Average of poorest ten cows.

Calculate the per cent of profit:

66. Average cow.
67. Best cow.
68. Poorest cow.
69. Average of best ten cows.
70. Average of poorest ten cows.

Find the per cent of butter fat:

71. Average cow.
72. Best cow.
73. Poorest cow.
74. Average of best ten cows.
75. Average of poorest ten cows.

Weight of Animals

76. A stock farmer of the Middle West gives the following figures on the cost of feeding a steer for a year. Find the total cost of feeding the steer a year.

Silage, 5 tons	@	$2.50	
Pasturage			$5.00
Corn, 20 bushels	@	$.60	
Labor			$2.00

77. A steer weighing 700 lb. is bought at $.07 per pound and fed for a year for $24.50. What is the amount of the investment at the end of the year?

78. If a steer after being fed for a year for $24.50 gains in weight from 700 lb. to 1400 lb., find the profit if the steer is sold at $.09½ a pound.

NOTE. Hogs and poultry lose about 20% of the live weight in dressing, while sheep and cattle lose from 33⅓% to 50%.

79. What is the dressed weight of a crate of chickens weighing 240 lb.?

80. A 380-lb. live hog, purchased at $6.75 a hundred, is retailed at an average of 15¢ a pound. What is the profit?

81. Experiments at agricultural stations have shown that 500 lb. of corn fed to hogs increases their weight about

100 lb. If hogs are worth $10 per hundred dressed, does a farmer gain or lose by feeding them 360 bu. of corn worth 60 ¢ a bushel? (1 bu. corn weighs 56 lb.) How much?

82. When pork sells at $7 per hundred live weight, how much ought corn to bring in order that neither loss nor gain may result in feeding it to hogs, on the basis of problem 81?

83. A drove of hogs average 200 lb. in December, and are fed corn on the basis given in problem 81 until they weigh 300 lb. If pork sells at $6 per hundred, live weight, in December, and the corn is worth 50¢ per bushel, at what price per hundred should the 300-lb. hogs be sold to bring the same amount of money as in December?

84. If these hogs, which sold at $6 per 100 lb. in December, were sold at $6.25 per 100 lb., live weight, after being fed the corn, how much did the owner receive per hog for his labor in feeding and caring for the hogs after paying for the corn?

85. I sell 5 steers weighing 4950 lb., and some cows weighing 3460 lb. What is the total of the dressed beef produced by them, allowing a loss of 40 % in dressing?

86. A butcher bought a steer weighing 1250 lb. When killed and dressed, the marketable meat weighed 58 % of the live weight. If he paid $6 per hundred for the steer and sold 375 lb. of the meat at an average of 10 ¢ per pound, at what average price must he sell the remainder of the carcass, to make 25 % on his investment? What items of expense in conducting the business, besides the price paid for the steer, have to be considered in determining the net profit?

87. Hens average 150 eggs a year. Allowing 1080 eggs for hatching, what is the income from a yard of 120 hens, when eggs are 20¢ a dozen, and 1850 lb., live weight, of young chickens are sold at 15¢ a pound, dressed weight?

Potatoes

Reports from forty growers of potatoes in the state of Iowa showed the following items of expense in growing 1 acre of potatoes:

Rent, or interest, on land (valued at $100 per acre)	$6.00
Seed, 12 bu. @ $.60	
Treatment of seed	.50
Cutting (by hand)	.50
Plowing	2.00
Disking (twice) @ $.25	
Harrowing (5 times) @ $.13	
Planting	1.50
Cultivating (5 times) @ $.25	
Spraying (3 times)	5.00
Digging	5.00
Hauling to market (4 miles)	3.50
Extra hand labor (hoeing, etc.)	2.00

88. Find the total cost per acre.

89. The average yield per acre was 140 bushels. What was the cost of production per bushel?

90. What was the average cost per bushel for hauling the crop to market?

91. How much did the rent amount to for each bushel?

92. What was the cost per bushel for digging?

93. How much did the plowing, disking, harrowing, planting, cultivating, and spraying add to the cost per bushel?

94. The average yield per acre for the state of Iowa is 82 bushels. If these growers had raised only an average crop, what would have been the cost per bushel?

95. The growers received an average of $.71 per bushel for their potatoes. What was the profit per acre? per bushel?

The following table contains data received from twenty potato growers in a Western state, in 1911:

Grower	Price rec'd per Bushel	Av. Yield per Acre in Bushels	Cost of Production per Acre
1	$1.48	150	$31.10
2	.90	250	37.90
3	1.00	200	39.00
4	1.00	140	32.30
5	.50	275	42.00
6	.50	250	30.50
7	.55	215	40.60
8	.50	225	36.10
9	1.00	80	35.25
10	.60	140	28.50
11	.76	100	27.80
12	.45	176	33.70
13	1.60	125	39.30
14	.53	247	30.60
15	1.00	100	30.25
16	.80	100	31.00
17	.50	250	34.00
18	.40	250	48.45
19	1.00	125	22.95
20	.50	320	30.00

96. What was the gross return per acre for each grower?

97. Determine the net profit per acre in each case.

98. What was the profit per bushel for each grower?

99. What was the cost of production per bushel for each grower?

100. What was the average cost of production per acre?

101. What was the average yield per acre?

Hay in Mows and in Stacks

The only accurate method of determining the number of tons of hay in a stack or mow is by weighing. The amount, however, may be determined, approximately, by measurement.

In high stacks and deep mows:

Wild hay requires about 400 cu. ft. per ton.
Timothy hay requires about 500 cu. ft. per ton.
Clover hay requires about 600 cu. ft. per ton.

For stacks or mows less than 10 ft. high, add about 100 cu. ft. to each of the foregoing.

102. How many tons of clover are there in a mow 36 ft. by 40 ft. and 24 ft. deep?

103. How many tons of wild hay are there in a mow 6 ft. narrower and 2 ft. deeper than the foregoing?

104. A mow of clover is 11 ft. wide, 16 ft. long, and 9 ft. high. What is its value at $8.50 per ton?

105. How many tons of wild hay are there in a stack 40 ft. long, 20 ft. wide, and 44 ft. high?

106. A mow is 24 ft. by 30 ft., and contains 35 tons of clover. How high is it?

107. A haymow designed to hold 150 tons of clover is 60 ft. long and 24 ft. high. How wide must it be?

Fencing

Barbed wire averages about one pound per rod. It is sold in spools containing from 95 pounds to 115 pounds.

108. What is the cost of fencing a square 40-acre tract with a 4 barbed wire fence at $2.85 per hundred?

109. A field 80 rd. long and 40 rd. wide is fenced with woven wire 32 in. wide at 30¢ a rod, and 2 barbed wires at $3 per hundred. The posts are 1 rd. apart and cost $12\frac{1}{2}$¢ apiece. What is the price of the material?

110. An 80-acre farm of two adjoining square forties is to be surrounded by a fence, and to be separated into 20-acre fields by cross fences. What is the cost of the material if the same kind is used as given in problem 109?

111. What is the cost of material required for surrounding a 40-acre square farm with a 4-wire fence, at \$2.75 per hundred, posts one rod apart at 15¢ apiece?

112. A man can drive 20 posts per hour and string 4 wires across 10 posts per hour. At \$2 per day of 8 hours, what is the cost of labor on the fence mentioned in problem 111?

Silos

A **silo** is a structure used for preserving in its green state finely cut corn stalks, sugar cane, alfalfa, etc. This finely cut vegetable material is called **silage**, or **ensilage**. Silos are built of wood, stone, brick, or cement. The circular silo is the most economical form.

The average weight of silage in silos from 25 ft. to 36 ft. high is about 40 pounds a cubic foot. Forty pounds is also the amount of silage ordinarily fed to a cow per day.

113. 10 A. of good corn should provide silage for 20 cows for 180 da. If 40 lb. a day are allowed for each cow, how many tons of silage should be furnished by 10 A.?

114. On this basis, how many acres of good corn are required to fill a silo 16 ft. in diameter and 36 ft. high?

115. How many head of stock will this supply for 200 days?

116. A circular silo 20 ft. in diameter contains 200 tons of ensilage. If this averages 40 lb. per cubic foot, find the depth of the silo.

117. If a silo is filled at the rate of 8 tons per day, how long will it take to fill a silo 16 ft. in diameter and 40 ft. high?

118. How many cows can be supplied from it for 190 days?

Water in Plant Growth

Experiments conducted by the Department of Agriculture have proved that about 300 lb. of water are required to mature 1 lb. of corn. Oats need 500 lb., while clover, barley, and wheat need about 400 lb. each to mature 1 lb. of the product.

119. How much water is required to mature 40 acres of oats averaging 45 bu. per acre? (1 bu. of oats weighs 32 lb.)

120. To what depth would it cover the field if applied at one time? (1 cu. ft. of water weighs 62.5 lb.)

121. A 10-acre field of corn averaging 100 bu. per acre receives 6 in. of rain. If the rest is supplied by irrigation, how many cubic feet of water are required, no allowance being made for evaporation? (70 lb. = 1 bu. of corn on cob.)

122. From a given strip of land with a firm surface, 5.34 inches of water evaporated during a given time. From an adjacent strip of like size and soil, whose surface was kept well tilled to a depth of 3 inches, only .88 inch evaporated in the same time. How many tons of water were saved the soil by tilling 80 acres?

123. How much less rainfall is needed on a 40-acre piece of corn kept well tilled than on a field of the same size that is allowed to remain hard and firm? What will be the weight in tons?

124. Find the difference between the amount of water required to mature a 10-acre field of corn yielding 90 bu. per acre and a field of wheat of the same size yielding 32 bu. per acre. Make no allowance for evaporation. (1 bu. wheat weighs 60 lb.)

125. What is the average rainfall in your state per year? How much would the total fall for one year on an acre weigh?

Miscellaneous

126. The value of properly cured and stored corn stover averages about $10 per ton. The yield per acre averages about 2 tons. A 20-acre field of corn yields 40 bu. of corn per acre. If the corn is worth 56¢ per bushel, what is the entire value of the crop?

127. Of a quantity of seed corn used for planting, 80 kernels out of 100 germinate. The stand of corn is what proportion of what it should be if every kernel germinated?

128. If 85 kernels out of 100 germinate, the stand of corn is what proportion of a perfect stand?

129. If 99% of seed corn planted in a certain field germinates and the yield is 36 bu. per acre, what would have been the yield had all the seed corn germinated?

130. If corn shrinks 8% in weight between October and June, and sells at 60¢ per bushel in June, how much should it sell for in October, to bring the same amount of money from one acre yielding 50 bu., October weight?

131. If the same corn sells for 55¢ per bushel in October, how much should it sell for per bushel in June, in order to yield 6% more than in October?

132. Potatoes shrink an average of 1% a month for 5 months, after being taken from the ground. In September, a farmer harvests a crop of 500 bu. of potatoes. The market price at that time is $1.00 per bushel. At what price per bushel must he sell them 5 months later, to repay him for the interest lost at 6%?

133. A farmer sells 100 bu. of potatoes on October 1 at $1.00 per bushel. If he had kept them until March 1 and sold them for $1.20 per bushel, how much would he have gained, reckoning interest at 6%?

134. If 10% of the potatoes in problem 133 were lost because of decay, would the farmer have gained or lost, and how much, by holding the potatoes?

135. The number of bushels of new corn in a crib equals $\frac{1}{6}$ the number of cubic feet of corn in the crib. The number of bushels of old corn in a crib equals $\frac{1}{4}$ the number of cubic feet of corn in the crib. A corn crib is 20 ft. long, 6 ft. wide at the bottom, 8 ft. wide at the top, and 6 ft. high. How many bushels of new corn will it hold? of old corn?

136. A corn crib 24 ft. long, 8 ft. wide at the bottom, 10 ft. wide at the top, and 7 ft. high, holds how many bushels of new corn? of old corn?

137. A corn crib has an average width of 7 ft. It is 8 ft. high. What must be its length to hold 250 bu. of new corn? How many bushels of old corn will it hold?

138. A farmer grows 800 bu. of corn which he sells for 65¢ per bushel. By giving one extra day to selecting and caring for his seed corn, he increases his yield the next year 4%. If corn sells at the same price as the year before, how much does he receive for his extra day's work?

139. Investigations made in a certain district showed that there was a loss of 10% in the oat crop because of the presence of smut. What was the loss to a farmer who harvested 450 bu. of oats that year, if oats sold for 48¢ per bushel?

140. The farmer mentioned in the preceding problem could have treated his seed oats with a formaldehyde solution at a cost of $2 for material and labor, thus preventing the loss by smut. How much would he have gained by so doing?

MANUFACTURING AND FREIGHT PROBLEMS

1. In 1850 there were 123,025 manufacturing establishments in the United States, and in 1900, 512,254. Find the per cent of increase for 50 years.

2. In 1850 the value of manufactured products in the United States was $1,019,106,616, and in 1900, $13,004,400,143. What per cent of increase was there from 1850 to 1900?

3. If raw silk loses 24% in being boiled, how much will 5000 lb. weigh after being boiled?

4. The value of manufactured products in the city of New York in 1904 was $206,825,000, and in 1909, $266,034,000. What per cent did the value increase in 5 years?

5. In 1913 $381,032,495 of the foreign carrying trade was in American vessels, and $3,391,998,429 in foreign vessels. What per cent of the total trade was in American vessels?

6. In 1913 the freight products of agriculture in the United States amounted to 85,566,053 tons; of animals, to 23,763,262 tons; of mines, to 539,255,980 tons; of forests, to 108,506,272 tons; of manufactures, to 135,175,536 tons; of merchandise, to 36,519,321 tons; of other commodities, to 38,447,567 tons. Find the total freight product. Find the per cent each product was of the total.

VALUES OF PRINCIPAL MANUFACTURES EXPORTED FROM THE UNITED STATES IN 1913

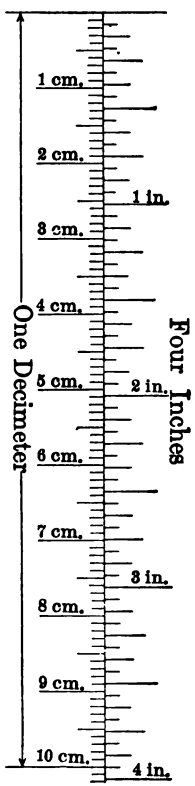
Iron and steel	$304,605,797	Chemicals, etc.	$26,574,519
Copper	$140,164,913	Leather	$63,893,351
Agricult. Implements	$ 40,572,352	Cotton	$53,743,977
Wood Manufactures	$115,704,777	Books, etc.	$10,092,719
Mineral oils	$129,666,995	Paper	$11,686,584

7. Find the total value of these manufactures. Find what per cent each value is of the total.

SUPPLEMENT

To the Teacher. The following subjects were omitted from the body of the book, because they have not much practical value. They are included in this supplement in case it is desired to teach them in some schools.

METRIC SYSTEM OF MEASURES



The system of weights and measures in our country is irregular and somewhat difficult to learn, as well as inconvenient to apply. The same held true with the old systems of other nations. In order to overcome these defects, France in 1799 adopted the metric system of weights and measures, based upon the decimal system of notation. This system is now in use in all civilized nations, England and the United States being the only exceptions. Congress in 1866 legalized its use in our country, but it is used here at the present time in scientific work chiefly.

Multiples of the primary units are represented by the Greek prefixes, **deka** (10), **hekto** (100), **kilo** (1000), **myria** (10,000); and decimal divisions, by the Latin prefixes, **deci** ($\frac{1}{10}$), **centi** ($\frac{1}{100}$), **milli** ($\frac{1}{1000}$).

LINEAR MEASURE

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer** (Km.)
10 kilometers	= 1 myriameter (Mm.)

The units most commonly used are those printed in black type; namely, the millimeter, centimeter, meter, and kilometer.

A decimeter, or 10 centimeters, as shown in the cut, equals a little less than 4 in. The meter is equal to 39.37 in., or 1.09 yd.

SQUARE MEASURE

100 sq. millimeters (sq. mm.)	= 1 sq. centimeter (sq. cm.)
100 sq. centimeters	= 1 sq. decimeter (sq. dm.)
100 sq. decimeters	= 1 **sq. meter** (sq. m.)
100 sq. meters	= 1 sq. dekameter (sq. Dm.)
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)
100 sq. hektometers	= 1 **sq. kilometer** (sq. Km.)

LAND MEASURE

100 centares (ca.) or square meters	= 1 **are** (a.), or 1 sq. Dm.
100 ares	= 1 **hektare** (Ha.)

MEASURES OF VOLUME

1000 cubic millimeters (cu. mm.)	= 1 **cubic centimeter** (cu. cm.)
1000 cubic centimeters	= 1 **cubic decimeter** (cu. dm.)
1000 cubic decimeters	= 1 **cubic meter** (cu. m.), or **stere**

MEASURES OF WEIGHT

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 **gram** (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 **kilogram** (Kg.)

MEASURES OF CAPACITY

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 **liter** (l.)

APPROXIMATE VALUES

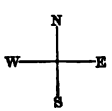
Metric Units	Approximate Equivalents
1 kilometer	$\frac{6}{10}$ mi.
1 meter	1.1 yards
1 centimeter	0.4 inch
1 hektare	2.5 acres
1 square meter	1.2 square yards
1 square centimeter	0.16 square inch
1 cubic meter	1.3 cubic yards
1 cubic centimeter	0.06 cubic inch
1 hektoliter	3 bushels
1 liter	0.9 quart (dry)
1 liter	1.06 quarts (liquid)
1 metric ton	1 long ton, nearly
1 kilogram	2.2 pounds
1 gram	15.4 grains

LAND SURVEY

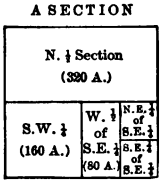
In most states the land is divided by survey into townships 6 miles square, and these are divided into 36 equal sections of a square mile, or 640 acres each. The sections are subdivided into half-sections and quarter-sections, and are numbered and designated as in the following diagrams.

A TOWNSHIP

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36



A SECTION



The townships are numbered and designated east or west from a definite north and south line, called the **principal meridian**, and north or south from a definite east and west line called the **base line.**

PROBLEMS

ORAL

1. How many acres are there in the N.E. $\frac{1}{4}$ of S.E. $\frac{1}{4}$ of section 18?

2. How many acres are there in the N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of section 34? in the W. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ of section 30?

3. Find the cost of the S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of section 9, at \$40 an acre.

4. How many rods of fence are required to inclose the land mentioned in problem 3?

5. Find the cost of the N. $\frac{1}{2}$ of S.E. $\frac{1}{4}$ of section 36, at \$50 an acre.

6. At \$100 an acre, find the cost of the S. $\frac{1}{2}$ of section 3 and the N.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ of the same section.

LONGITUDE AND TIME

Circular measure is used in measuring arcs of circles and angles, and in estimating latitude and longitude.

TABLE

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (deg. or °)
360 degrees	= 1 circumference

The **longitude** of a place is its distance east or west from a given meridian. The meridian most commonly used for this purpose is the one passing through the observatory at Greenwich, England. It is called the prime meridian. All places on it have 0° longitude. Longitude is reckoned in degrees, minutes, and seconds, and ranges from 0° to 180° east or west. The earth revolves on its axis from

west to east once in every 24 hours, and this causes the sun to appear to revolve around the earth from east to west in the same time. Since the sun travels through the entire circumference, or 360°, in twenty-four hours, in one hour it travels $\frac{1}{24}$ of 360°, or 15°; in one minute it travels $\frac{1}{60}$ of 15°, or 15′; and in one second it travels $\frac{1}{60}$ of 15′, or 15″.

Places to the *east* of any meridian have *later* time; those to the *west*, have *earlier* time, since the sun appears first to those in the east. When the sun is directly overhead, it is noon on that meridian.

Table of Longitude and Time

A difference of 15° of longitude makes a difference of 1 hour of time.
A difference of 15′ of longitude makes a difference of 1 minute of time.
A difference of 15″ of longitude makes a difference of 1 second of time.

PROBLEMS

Oral and Written

1. How many degrees of longitude make a difference of 1 hour in time? 12 hours? 30 minutes? 20 seconds?

2. When it is noon at Chicago, what is the time 15° east? 15° west? 30° east? 30° west? 60° east? 60° west?

3. What is the difference in longitude between two cities, if the difference in time is 2 hours? 3 hours 30 minutes?

4. If I start at Denver and travel until my watch is 1 hr. 30 min. too fast, in what direction and how far do I travel?

5. The first shock of the San Francisco earthquake, April 18, 1906, was recorded at 5.12 A.M. in observatories in 120° W. and at 19 min. 20 sec. past 8 A.M. in 75° W. What length of time was consumed by the shock in passing between the two observatories?

6. The longitude of Philadelphia is 75° 10′ 0″ west, and of Vienna 16° 20′ 22″ east. What is the time in Vienna when it is 9 A.M. in Philadelphia?

$$\begin{array}{rrr} 16° & 20′ & 22″ \text{ E.} \\ 75° & 10′ & 0″ \text{ W.} \\ \hline 15)91° & 30′ & 22″ \end{array}$$

6 hr. 6 min. $1\frac{7}{15}$ sec.

9 A.M. + 6 hr. 6 min. $1\frac{7}{15}$ sec. =

3 hr. 6 min. $1\frac{7}{15}$ sec. P.M. *Ans.*

Since one place is in east, and the other in west, longitude, the difference in longitude is found by *adding*, which gives 91° 30′ 22″. From the table on p. 475 it follows that the difference in time is as many hours, minutes, and seconds, respectively, as there are degrees, minutes, and seconds in $\frac{1}{15}$ of the difference in longitude, or 6 hr. 6 min. $1\frac{7}{15}$ sec. Since Vienna is in east longitude, and Philadelphia in west longitude, the sun rises earlier in Vienna; hence we must add the difference in time to 9 A.M., which gives 6 min. $1\frac{7}{15}$ sec. past 3 P.M.

7. The difference in longitude of two places is 40°. What is the difference in time between the places?

8. The longitude of New Orleans is 90° west. Find the difference in time between this place and Philadelphia, which is in longitude 75° 10′ 0″ west.

9. London is on the prime meridian, and Washington is 77° 3′ 6″ west. What change must a traveler make in his timepiece in going from Washington to London?

10. Paris is 2° 20′ east longitude, and San Francisco 122° 26′ 15″ west. What time is it in San Francisco when it is 7 P.M. in Paris?

11. Rome is 12° 27′ east, and Manila 120° 58′ 6″ east. Dewey began firing on the defenses of Manila at 5.41 A.M., May 1, 1898. At what time, by timepieces in Rome, was the fight begun?

12. The difference in time between Philadelphia and Cincinnati is 37 min. 20 sec. What is the difference in longitude?

37 min. 20 sec.
15
9° 20′ 0″. *Ans.*

Since a difference of 1 minute of time corresponds to a difference of 15′ of longitude, etc., 15 times the number of seconds of time will represent seconds of longitude (300″), and 15 times the number of minutes of time, the number of minutes of longitude (555′), 555′ + 300″ = 9° 20′ 0″.

13. The time at St. Louis is 53 min. earlier than the time at Washington. What is the difference in longitude?

14. In traveling from New York to Cincinnati I find that my watch is 41 min. 32 sec. too fast. Find the difference in longitude.

15. In traveling on a parallel of latitude from 89° 32′ 30″ west longitude, I notice that my watch has lost 1 hr. 25 min. 30 sec. In what direction am I traveling, and what is the longitude of the second place?

16. The longitude of New York is 74° 3′ west, and of Jerusalem 35° 13′ east. When it is 4.30 A.M. in New York, what is the time in Jerusalem?

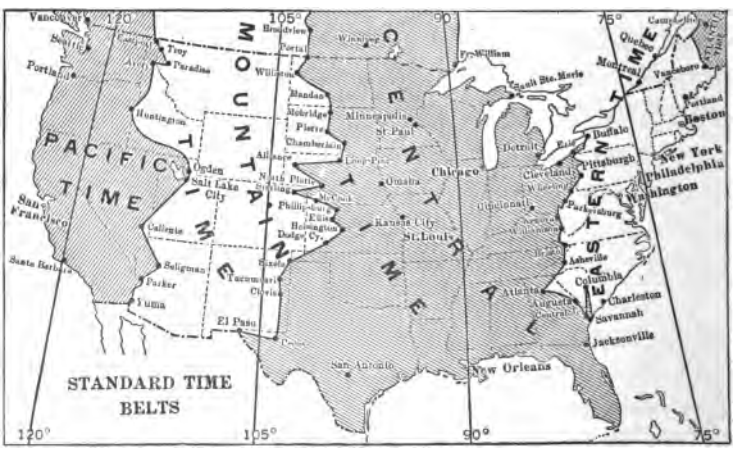
17. Formulate a problem in longitude and time, getting the data from your geography. Solve the problem.

STANDARD TIME

In the work on longitude and time, it was shown that all places differing in longitude differ in time as reckoned by the sun. In order to avoid the confusing differences in time when each place has its own local time, the railroads of Canada and the United States in 1883 formulated a system

of **standard time.** In this system, meridians that are multiples of 15° are selected as principal meridians, and the belts $7\frac{1}{2}$° on either side have the local time of the principal meridian. In practice, however, the time belts are not of uniform width, since the division terminals of railroads often extend beyond, or fall short of, the halfway points between the principal meridians. The time belts used in standard time are the Atlantic, Eastern, Central, Mountain, and Pacific, with principal meridians of 60°, 75°, 90°, 105°, and 120°, respectively.

A traveler leaves New York at 8 A.M. As he leaves Pittsburgh, he notices that his watch, an accurate timekeeper, is an hour fast. Pulling into Ellis, Kansas, his watch is still an hour fast, but as the train leaves the station he notices that his watch is two hours fast. When leaving Ogden, Utah, he notices that his watch has gained another hour. His timepiece now is three hours fast, and remains so during the rest of the trip to San Francisco. Explain.



SIMPLE INTEREST

TABLE METHOD

People who have much interest to compute generally use **interest tables.** The following partial table is based on a year of 360 days and a rate of 6 %.

Years	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	60	120	180	240	300	360	420	480	540
2	120	240	360	480	600	720	840	960	1080
3	180	360	540	720	900	1080	1260	1440	1620
4	240	480	720	960	1200	1440	1680	1920	2160
5	300	600	900	1200	1500	1800	2100	2400	2700
6	360	720	1080	1440	1800	2160	2520	2880	3240
7	420	840	1260	1680	2100	2520	2940	3360	3780

Months	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	5	10	15	20	25	30	35	40	45
2	10	20	30	40	50	60	70	80	90
3	15	30	45	60	75	90	105	120	135
4	20	40	60	80	100	120	140	160	180
5	25	50	75	100	125	150	175	200	225
6	30	60	90	120	150	180	210	240	270
7	35	70	105	140	175	210	245	280	315
8	40	80	120	160	200	240	280	320	360

Days	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	.167	.333	.50	.667	.833	1.00	1.167	1.133	1.50
2	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00
3	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50
4	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00
5	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50
6	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
7	1.167	2.333	3.50	4.667	5.833	7.00	8.167	9.333	10.50
8	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00
9	1.500	3.000	4.50	6.000	7.500	9.00	10.500	12.000	13.50
10	1.667	3.333	5.00	6.667	8.333	10.00	11.667	13.333	15.00
11	1.833	3.667	5.50	7.333	9.167	11.00	12.833	14.667	16.50
12	2.000	4.000	6.00	8.000	10.000	12.00	14.000	16.000	18.00
13	2.167	4.333	6.50	8.667	10.833	13 00	15.167	17.333	19.50
14	2.333	4.667	7.00	9.333	11.667	14.00	16.333	18.667	21.00
15	2.500	5.000	7.50	10.000	12.500	15.00	17.500	20.000	22.50

WRITTEN

1. Find by the table on page 479 the interest on $3560 for 5 yr. 6 mo. 6 da. at 6%.

NOTE. The interest on $500 is $\frac{1}{10}$ of that on $5000; the interest on $60 is $\frac{1}{100}$ that on $6000.

SOLUTION.

Int. on $3000 =	$900 +	$90 +	$3.00	
Int. on 500 =	150 +	15 +	.50	
Int. on 60 =	18 +	1.80 +	.06	
Int. of $3560	$1068 +	106.80 +	3.56	= $1178.36

Find by this method the interest, at 6%, on:

2. $2000, 1 yr. 1 mo. 5 da.
3. $1520, 2 yr. 3 mo. 6 da.
4. $3460, 3 yr. 2 mo. 7 da.
5. $5220, 4 yr. 4 mo. 10 da.
6. $8140, 5 yr. 5 mo. 12 da.
7. $7260, 6 yr. 8 mo. 15 da.
8. $6418, 3 yr. 5 mo. 8 da.
9. $5105, 2 yr. 4 mo. 6 da.

CANCELLATION METHOD

NOTE. At 6% the interest on any principal for 1 day is $\frac{1}{1000}$ of the principal divided by 6. This result should be multiplied by the number of days.

1. Find the interest on $4268 for 63 days (*a*) at 6%; (*b*) at 4%.

SOLUTION. (*a*) $\$2.134 \times \frac{21}{\cancel{63}} = \$44.814$;

(*b*) $\$2.134 \times \frac{7}{\cancel{21}} \times \frac{2}{\cancel{4}} = \$29.876$

Find by the cancellation method the interest of:

2. $5000, 15 da., 6%.
3. $6000, 10 da., 4%.
4. $4500, 60 da., 5%.
5. $7200, 90 da., 3%.

COMPOUND INTEREST

Amount of $1 *at various rates, compound interest,* 1 *to* 17 *years*

YEARS	1 PER CENT	1½ PER CENT	1¾ PER CENT	2 PER CENT	2½ PER CENT	3 PER CENT
1	1.010000	1.012500	1.015000	1.020000	1.025000	1.030000
2	1.020100	1.025156	1.030225	1.040400	1.050625	1.060900
3	1.030301	1.037971	1.045678	1.061208	1.076891	1.092727
4	1.040604	1.050945	1.061364	1.082432	1.103813	1.125509
5	1.051010	1.064082	1.077284	1.104081	1.131408	1.159274
6	1.061520	1.077383	1.093443	1.126162	1.159693	1.194052
7	1.072135	1.090850	1.109845	1.148686	1.188686	1.229874
8	1.082857	1.104486	1.126493	1.171659	1.218403	1.266770
9	1.093685	1.118292	1.143390	1.195093	1.248863	1.304773
10	1.104622	1.132271	1.160541	1.218994	1.280085	1.343916
11	1.115668	1.146424	1.177949	1.243374	1.312087	1.384234
12	1.126825	1.160755	1.195618	1.268242	1.344889	1.425761
13	1.138093	1.175264	1.213552	1.293607	1.378511	1.468534
14	1.149474	1.189955	1.231756	1.319479	1.412974	1.512590
15	1.160969	1.204829	1.250232	1.345868	1.448298	1.557967
16	1.172579	1.219889	1.268986	1.372786	1.484506	1.604706
17	1.184304	1.235138	1.288020	1.400241	1.521618	1.652848

YEARS	3½ PER CENT	4 PER CENT	4½ PER CENT	5 PER CENT	6 PER CENT	7 PER CENT
1	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.272279	1.315932	1.360862	1.407100	1.503630	1.605782
8	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.511069	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.563956	1.665074	1.772196	1.885649	2.132928	2.409845
14	1.618695	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.675349	1.800944	1.935282	2.078928	2.396558	2.759032
16	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.794676	1.947901	2.113377	2.292018	2.692773	3.158815

Compound interest tables are used in computing compound interest. The table on page 481 shows the amount of $1 at compound interest for any number of years up to 17, and for any rate per cent up to 7.

WRITTEN

1. Find, by using the table, the amount of $500 for 10 years at 7 %, compound interest.

SOLUTION

The amount of $1 for 10 years at 7 %, as shown by the table (p. 481), equals $1.967151.

Hence the amount of $500 = 500 × $1.967151, or $983.58. *Ans.*

2. Find the amount of $800 for 12 years at 6 %, compound interest.

3. What is the compound interest on $2000 for 15 years at 7 %?

4. Find the amount of $1200 for 8 years at 4 %, compounded semiannually.

NOTE. When interest is compounded semiannually, one half the given rate and twice the number of periods for which interest is compounded are used. Thus, in problem 4, find the interest at 2 % for 16 years.

5. What is the amount and the compound interest of $400 for 2 years, 3 months at 6 %, interest compounded semiannually?

NOTE. Compute the interest for 4 years at 3 %, and then for the remaining time at 6 %.

6. Find the compound interest on $5000 for 6 years at 7 %, compounded semiannually.

7. Find the compound interest on $10,000 for 3 years at 6 %, compounded semiannually.

PARTIAL PAYMENTS

Sometimes the maker of a note is not able to pay it in full at maturity, or is allowed, if he chooses, to make part payment before the note is due. Such payment is then indorsed on the back of the note with the date of payment. Several such payments may be made before the entire amount is paid. Such payments are called **partial payments.**

The following problem is solved by the rule adopted by the United States Supreme Court, called the *United States Rule.* It is based on the following principles:

(1) *Payments are applied first to discharge or pay the interest due on the principal at the time the payment is made, and the balance, if any, is then applied to reduce the principal.*

(2) *Only the principal draws interest; hence, if the payment is less than the interest due when it is made, the unpaid interest must not be added to the principal and thus draw interest. The interest is then calculated to the next period before the payment is deducted.*

1. What amount is due on the following note:

> *$2000* *South Bend, Ind., Feb. 1, 1914.*
>
> *On demand, for value received, I promise to pay Arthur Carter ~~~~~ or order, Two thousand and 00/100 ~~~~~ Dollars.*
>
> *With interest at 5%.* *Frank Morse.*

The following payments are indorsed on this note:

Aug. 1, 1914	$100	Nov. 1, 1915	$100
Aug. 1, 1915	50	Find balance due Feb. 1, 1916.	

SOLUTION

Principal	$2000
Int. on $2000, Feb. 1, 1914, to Aug. 1, 1914 (6 mo.)	50
Amt. of $2000, Aug. 1, 1914	2050
Payment, Aug. 1, 1914	100
New principal, Aug. 1, 1914	1950
Int. on $1950, Aug. 1, 1914, to Aug. 1, 1915 (1 yr.) $97.50. As the interest exceeds the payment of $50 made Aug. 1, 1915, no new principal is formed	
Int. of $1950, Aug. 1, 1914, to Nov. 1, 1915 (1 yr. 3 mo.)	121.88
Amt. of $1950, Nov. 1, 1915	2071.88
Less payments, $50 (Aug. 1, 1915) and $100 (Nov. 1, 1915)	150
New principal, Nov. 1, 1915	1921.88
Int. of $1921.88, Nov. 1, 1915, to Feb. 1, 1916 (3 mo.)	24.02
Amt. due, Feb. 1, 1916	$1945.90

2. A note of $1000, dated March 1, 1914, with interest at 4 %, is indorsed as follows: June 1, 1914, $90; March 1, 1915, $500. How much is due on the note March 1, 1916?

3. On a note for $5000, dated July 1, 1913, with interest at 6 %, the following indorsements are made: July 1, 1914, $40; July 1, 1915, $500. How much is due July 1, 1916?

When notes are fully paid within a year from date, the following rule, called the *Merchants' Rule*, is ofted used.

Find the amount of the principal from the time it begins to draw interest to the time of final settlement.

Find the amount of each payment from the time it is made to the date of settlement.

From the amount of the principal subtract the sum of the amounts of the payments. The difference is the balance due.

4. John Smith gives Henry Jones his note for $250, due in one year from Dec. 15, 1913, interest at 7 %.

On March 15, 1914, Smith makes a payment of $75, and on Sept. 15, 1914, he makes another payment of $50. How much is due at maturity?

SQUARE ROOT

Explanation of the Process

The square of any number composed of tens and units equals the square of the tens plus twice the product of the tens by the units plus the square of the units.

Or, substituting t for tens and u for units, $(t + u)^2 = t^2 + 2\,tu + u^2$.

1. Extract the square root of 625.

```
 2 5
6'25
 4 00
40 2 25
   2 00
     25
     25
```

Beginning at units to point off into periods of two figures, you find that the first period consists of one figure, 6.

It has been shown that the square of the first figure of the root is found in the first period. The greatest square equal to or less than 6 is 4. Its square root is 2, the first figure of the root. The two periods show that there are two figures in the root, hence the value of its first figure in units is 20. The square of 2 tens, or 20, is 4 hundreds, or 400. To show clearly what is done, the two zeros are annexed to 4, the square of the first figure of the root, and the subtraction is then made. The result, 225, is the same as though the zeros had been omitted (as is ordinarily done), the 4 subtracted from the first period and the next period annexed to the remainder.

You have now taken from the square one of the three parts of which it is composed; namely, the square of the tens. The remainder is made up of the other two parts; namely, 2 times the product of the tens by the units + the square of the units ($2\,tu + u^2$).

You have found the tens' figure of the root, but have yet to find the units' figure. Of the remaining two parts, the second part, the square of the units is much the smaller, and in finding the trial units' figure may be ignored. Leaving it out of account, you may regard the 225 as made up of 2 times the product of the tens, or 20, by the units. Here you have the product of three factors, and two of them known, to find the other factor. The two known factors in 2 times the product of 20 by the units are 2 and 20. To find the trial units' figure in the root, divide 225, the supposed product of the three factors, by 40, the product of the known factors, 2 and 20, and you have 5 for the quotient, or the other factor, the units' figure of the root.

Multiplying 40 by 5, you have 200, or 2 times the product of the tens by the units, or the second part of the square. Subtracting, you have left 25, or the square of the units, the third part of the square. If the trial units' figure is correct, its square equals 25, which is true; hence 5 is the correct units' figure.

NOTE. The square subtracted from the first period, and the product of the second figure of the root by the complete divisor, together equal the entire number. If there were three figures in the root, these two numbers together would equal the square of the first two figures of the root considered as tens, and the units' figure would remain to be found in the same way as the second figure of the root was found, and so on, if the root contained more than three figures.

```
 2 5
 625
   4
45|225
  |225
```

In actual work all the zeros are omitted. The trial divisor, 4, is used to divide the dividend, 225, exclusive of the right-hand figure, or to divide 22, as this gives the same result as though 40 and 225 were used. When the trial units' figure is found, it is annexed to 4, the trial divisor, making 45. 45 is equal to 2 times the tens + 5. This being multiplied by 5, the units' figure, gives 2 times the product of the tens by the units + the square of the units in one operation.

2. Extract the square root of 3249, explaining the process.

CUBE ROOT

$1^3 = 1$	$4^3 = 64$	$7^3 = 343$
$2^3 = 8$	$5^3 = 125$	$8^3 = 512$
$3^3 = 27$	$6^3 = 216$	$9^3 = 729$

1. Extract the cube root of 12,167.

```
                2 3
              12'167
                 8
3 × 20²  = 1200 4167
3 × 20 × 3 =  180
     3²  =    9 4167
            1389
```

Steps:

1. Separate the number (12,167) into periods of three figures each, beginning with units.

2. Find the greatest cube (8) equal to or less than the left-hand period (12); write its cube root (2) for the first figure of the root and subtract the cube (8) from the left-hand period (12).

3. To the remainder (4) annex the next period (167) for the dividend.

4. Write three times the square of the first figure (2) of the root, considered as tens (or 20), as trial divisor (1200), at the left of the dividend (4167).

5. Divide the dividend (4167) by the trial divisor (1200) and write the quotient for the next figure of the root.

6. Add to the trial divisor three times the first part of the root (2) considered as tens (or 20), multiplied by the last figure (3), (180); and also the square of the last figure (3) of the root (9), to make the complete divisor (1389).

7. Multiply the complete divisor (1389) by the last figure (3) of the root, and subtract the product from the dividend (4167).

8. Repeat steps 3 to 7 if there are more than two figures in the root, substituting the proper numbers for those given.

NOTES. 1. If at any time the trial divisor is not contained in the dividend, place a zero in the root, annex two zeros to the trial divisor, and annex the next period to the dividend and then divide.

2. If at any time the product of the complete divisor by the last figure of the root found is greater than the dividend, the root figure is too large. In that case reduce the root figure by 1, make the new additions to the trial divisor, and proceed as before.

3. In extracting the cube root of decimals, point off from the decimal point to the right. If necessary, annex zeros, so that each period may have three places.

4. In finding the cube root of a fraction whose terms are not perfect cubes, reduce the fraction to a decimal with three times as many places as the number of figures wanted in the root, and extract the root.

TO THE TEACHER. The statement of the steps 1 to 8, omitting the numbers, is the rule for extracting the cube root. Each step should be thoroughly learned. Pupils should be able to state the entire rule and to apply each step in order. After going through the rule with pupils to see that they understand and can apply its directions, give drills in applying each step to a number of examples before taking up the next step. Before this drill is begun on any step, have the exact wording of the step learned and then apply it without referring to the book.

Point off the following numbers for the extraction of their cube roots, keeping the work on each number on a separate sheet of paper:

2.	3.	4.	5.	6.
32768;	79507;	614125;	373.248;	.004913

State the first step.

Give the cubes of the numbers from 1 to 10.

State the second step, and apply it in each case.

State the third step, and apply it to each remainder.

In the same way, state and apply the succeeding steps with each of the results thus far obtained.

Explanation of the Process

Relation of the number and places of figures in a root to the number and places of figures in its cube.

$1^3 = 1$ The cube of the smallest number of one figure consists of one figure.

$9^3 = 729$ The cube of the largest number of one figure consists of three figures.

The cube of any number between 1 and 9 contains one, two, or three figures.

$10^3 = 1000$
$90^3 = 729{,}000$ Annex a zero to 1, making 10, and to 9, making 90, and cube these numbers. Notice that annexing a zero to the smallest and to the largest number of one figure increases the number of figures in the cube by three.

$19^3 = 6859$
$99^3 = 970{,}299$ Annex 9, the largest number of one figure, to 1, making 19, and to 9, making 99. It will be seen that annexing the figure 9 to the smallest and to the largest number of one figure increases the number of figures in each cube by three. Hence, annexing a figure of any value to any one-figure number increases the number of figures in its cube by three.

$10^3 = 1\,000$
$100^3 = 1\,000\,000$
$900^3 = 729\,000\,000$
$990^3 = 970\,299\,000$
$999^3 = 996\,002\,999$

In like manner, it may be shown that annexing a figure to any number increases the number of figures in its cube by three.

The cube of the first figure of the root is found in the first period of the cube; of the first two figures of the root, in the first two periods of the cube; of the first three figures of the root, in the first three periods of the cube, etc.

From the preceding, it appears that the cube of any number of one figure contains one, two, or at most three figures, and that if any figure is annexed to any one-figure number, the number of figures in its cube is increased by three; that for any figure after the first in the root there are three figures in its cube, and that for the first figure in the root there may be one, two, or three figures in the cube; that pointing off the number into periods of three places each, beginning with units, will show the first period, whether of one, two, or three figures, and the number of figures in the root.

Composition of the Cube of a Number

A number is cubed by multiplying its square by the number.

$$25^2 = 20^2 + 2(20 \times 5) + 5^2$$

Multiply $(20 + 5)^2$ by $20 + 5$, indicating the operations. Remember that multiplying any one of several factors of a product multiples the product.

$$\begin{array}{r} 20^2 + 2(20 \times 5) + 5^2 \\ \hline 20 + 5 \\ \hline 20^2 \times 5 \quad + 2(20 \times 5^2) + 5^3 \\ 20^3 + 2(20^2 \times 5) \quad + (20 \times 5^2) \\ \hline 20^3 + 3(20^2 \times 5) \quad + 3(20 \times 5^2) + 5^3 \end{array}$$

In the partial products we have 2 times 20×5^2 and once 20×5^2; the sum is 3 times 20×5^2.

We also have once $20^2 \times 5$ and 2 times $20^2 \times 5$; the sum is 3 times $20^2 \times 5$.

$$25^3 = (20 + 5)^3 = 20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3.$$

Substituting t for the tens, or 20, and u for the units, or 5, we have:

$$(t + u)^3 = t^3 + 3(t^2 \times u) + 3(t \times u^2) + u^3.$$

$t^3 = 20^3 = 8000$ = Cube of the tens.

$3(t^2 \times u) = 3(20^2 \times 5) = 6000$ = Three times the square of the tens by the units.

$3(t \times u^2) = 3(20 \times 5^2) = 1500$ = Three times the tens by the square of the units.

$u^3 = 5^3 = 125$ = Cube of the units.

$$t^3 + 3(t^2 \times u) + 3(t \times u^2) + u^3 =$$
$$20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3 = 15{,}625.$$

As any number of two or more figures may be regarded as made up of a certain number of *tens* + a certain number of units, it follows that the cube of any number of two or more figures equals:

The cube of the tens plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

$$75^3 = (70+5)^3 = 70^3 + 3(70^2 \times 5) + 3(70 \times 5^2) + 5^3 = ?$$
$$215^3 = (210+5)^3 = 210^3 + 3(210^2 \times 5) + 3(210 \times 5^2) + 5^3 = ?$$

1. Extract the cube root of 15,625.

We have found that the cube root of a number is made up of four parts:

(1) The cube of the tens. (2) Three times the square of the tens multiplied by the units. (3) Three times the tens multiplied by the square of the units. (4) The cube of the units.

We have found also that the cube of the first figure of the root is found in the first period of the cube.

$$t^3 = \begin{array}{r|l} & 2\ 5 \\ \hline 15'625 & \\ 8 & \\ \hline 7\ 625 & \end{array}$$

$$\left.\begin{array}{r} 3(t^2 \times u) \\ 3(t \times u^2) \\ u^3 \end{array}\right\} \text{A}$$

The first period is 15. The largest cube equal to or less than 15 is 8; its cube root is 2, the first figure of the required root.

As there are two periods, and therefore two figures in the root, the first figure, 2, is 2 tens, or 20 units. As the cube 8 is in thousands' place, you need not write the three zeros.

In subtracting 8 thousand from 15,625, you take away the cube of the tens, and have left the other three parts of the cube, as shown at A.

The first of these parts, $3(t^2 \times u)$, is the largest of the three, and as it contains all the different kinds of factors found in the other two parts, that is, 3, t, and u, you may disregard the other two parts and use only the first of these parts in finding the trial second figure of the root.

This is called a trial figure because as the other two parts of the cube have been disregarded, and 7625 has been considered as made up of only one of the three parts, the trial divisor may be found to be too large when

the two parts disregarded for the purpose of finding the units' figures are considered.

Regarding 7625, then, as made up of 3 times the square of the tens multiplied by the units, you see that two of these factors are known, namely: 3, and the tens, which you have found to be 2, or 20 units. The square of the tens is 20×20, or 400. 3 times the square of the tens is 1200. The units' figure is not known, but may be found approximately by dividing 7625 by 1200 as a trial divisor, or, what is the same thing, ignoring the zeros and dividing 76 by 12. The quotient is 6, the trial units' figure.

$$\left.\begin{aligned} 3(20^2 \times 6) &= 7200 \\ 3(20 \times 6^2) &= 2160 \\ (6^3) &= \underline{\ 216} \\ &\ \ 9576 \end{aligned}\right\} \text{A} \qquad \left.\begin{aligned} 3(20^2 \times 5) &= 6000 \\ 3(20 \times 5^2) &= 1500 \\ 5^3 &= \underline{\ 125} \\ &\ \ 7625 \end{aligned}\right\} \text{B}$$

Before writing the 6 in the root, test to see whether it is the proper figure. Writing out the value of each of the three parts as shown at A, you find their sum is 9576, or more than 7625. Therefore the trial units' figure, 6, is too large. Substituting for it the next smaller number, 5, and writing out the value of the three parts as shown above at B, you find that their sum is 7625, showing that 5 is the correct units' figure, which you place in the root over the second period.

In the process employed in extracting the cube root of numbers, you have shortened the above operation as follows:

After dividing 7625 by 1200 as a trial divisor, and finding the trial units' figure, 6, you may complete the divisor by adding to it $3(20 \times 6)$, or 360, and 6^2, or 36, making $1200 + 360 + 36$, or 1596, which equals $3 \times t^2 + 3(t \times u) + u^2$. Each of these three parts lacks the same factor, the units.

		7625
Trial divisor, or		
$3 \times t^2$	1200	
$3 \times t \times u$	300	
u^2	25	7625
Complete divisor	1525	

On multiplying their sum, 1596, called the complete divisor, by 6, the product, 9576, is the same as you found the sum of the three full parts to be, in testing to determine whether 6 was the correct units' figure.

In testing 5, proceed in the same way: Adding to 1200, $3(20 \times 5)$, or 300, and 5^2, or 25, you have 1525, the complete divisor, which, when multiplied by the units' figure, 5, gives 7625, the sum of the three parts of the cube remaining after the cube of the tens, 8000, has been subtracted.

The cube subtracted from the first period, and the product of the complete divisor by the second figure of the root, together equal the entire number or cube. If there were three figures in the root, these two numbers together would equal the cube of the first two figures of the root considered as tens, and the units' figure would be found in same way as the tens' figure was found, and so on, if the root had more than three figures.

2. Extract the cube root of 97,336, explaining the process. Work the example, answering each question in order.

The information necessary for answering these questions may be obtained by a careful study of the examination of the process of extracting the cube root, pp. 488–492.

1. Why do you point off the number into periods of three figures each, beginning at units' place?
2. How is the first figure of the root found? Why? What is it?
3. The number subtracted from the first period is what as related to the first figure of the root?
4. The number formed by annexing the next period to the remainder contains what three parts of the entire cube?
5. What part of the entire cube is it regarded as being for the purpose of finding the second figure of the root?
6. What factors does this part contain?
7. Which of these factors are known?
8. When the product of three factors and two of the factors are known, how can you find the other factor?
9. What is done in this case to find the trial second figure of the root? What is this figure?
10. What is the trial divisor? Of what factors is it the product? What two additions are made to it to form the complete divisor? Give the numbers added, and state in general terms of what factors each number is the product.
11. State the terms of which the complete divisor is composed.
12. The product of the complete divisor by the trial units' figure of the root is composed of what terms of the cube?
13. What is done with this product? What does this operation show?

3. Extract the cube root of 148,877, explaining each step in the operation.

COMPOUND PROPORTION

A **compound proportion** is an equality of ratios, one or more of which are compound;

as, $4:8 = 3:6$
$2:10 = 5:25$

1. If 10 men can build 50 rods of wall in a certain time, how many men can build 80 rods of wall in the same time?

STATEMENT. $50:80 = 10:?$

2. If 10 men can build a wall in 12 days, how many men can build it in 16 days?

STATEMENT. $16:12 = 10:?$

The above problems are problems in simple proportion. Each statement is the statement of a simple proportion. Combining the two problems into one, you have the following problem:

3. If 10 men can build 50 rods of wall in 12 days, how many men can build 80 rods of wall in 16 days?

This problem is a problem in compound proportion. It will be noticed that in each of the two statements given above, the third term is 10. The first ratio in the first statement differs from the first ratio in the second statement. Bringing these statements together as the statement of a compound proportion, you have the third term 10, and the first and second terms, a compound ratio.

$$\frac{50:80}{16:12} = 10?$$

Multiplying the two ratios together, term by term, you have:

$$(50 \times 16):(80 \times 12) = 10:?$$

Multiplying the means together, and dividing by the extremes, you have:

$$\frac{80 \times 12 \times 10}{50 \times 16} = 12.$$

It will thus be seen that problems in compound proportion result from combining two or more problems in simple proportion.

In solving such problems, it is best to use the fractional form of solution. This gives the last expression above, without the preceding steps.

Notice that in the above problem there are two numbers representing rods, two numbers representing days, and one number representing men, and that what is asked for is the number of men. In getting the statement for multiplication, proceed as in simple proportion, using the third term first with the two numbers representing rods.

Q. If 10 men can build 50 rods of wall, will it take a greater or a less number to build 80 rods of wall in the same time? *Ans.* A greater number, or $\frac{80}{50} \times 10$ men.

Write the expression $\frac{80}{50} \times 10$ men at the right as follows: $\frac{80}{50} \times 10$.

Next deal with the men and days, disregarding everything else.

Q. If 10 men can build a wall in 12 days, will it take more or less men to build it in 16 days? *Ans.* Less men, or $\frac{12}{16} \times 10$ men.

Write this fraction, $\frac{12}{16}$, at the left of the expression above, so that it now stands:

$$\frac{12}{16} \times \frac{80}{50} \times 10 = 12.$$

Canceling and performing the indicated operations, you secure the result, 12.

4. If 18 men build 75 rods of fence in 10 days of 8 hours each, how many hours a day must 24 men work to build 144 rods in 12 days?

In this problem, hours a day are called for. You have two numbers, 18 and 24, representing men; two numbers, 75 and 144, representing rods; two numbers, 10 and 12, representing days; and one number, 8, representing hours. Each set of two like numbers constitutes a ratio, and must be compared with the number representing hours.

Q. If 18 men build a fence working 8 hours a day, must 24 men work more or less hours a day to build it in the same number of days? *Ans.* Less, or $\frac{18}{24}$ as many hours. $\frac{18}{24} \times 8.$

Q. If a number of men build 75 rods of fence working 8 hours a day, must they work a greater or less number of hours a day to build 144 rods in the same number of days? *Ans.* A greater number of hours a day, or $\frac{144}{75}$ as many hours. Write $\frac{144}{75}$ at the left of the expression above. $\frac{144}{75} \times \frac{18}{24} \times 8.$

Q. If it requires a number of men 10 days working 8 hours a day to build a fence, will it take a greater or less number of hours a day if they work 12 days? *Ans.* A less number, or $\frac{10}{12}$ as many hours. Prefix this fraction to the others with the multiplication sign. $\frac{10}{12} \times \frac{144}{75} \times \frac{18}{24} \times 8.$

5. If 37 horses in 5 days eat $35 worth of oats, how much will it cost to feed 60 horses on oats for 8 days?

Q. In this problem what is asked for? *Ans.* Dollars.

In the solution of these problems begin the question with the statement given in the problem.

If 37 horses eat $35 worth of oats in a certain time, will it cost more or less to feed 60 horses the same time? *Ans.* More, $\frac{60}{37}$ as much. Write this fraction at the left of 35, with the sign of multiplication between them. $\frac{60}{37} \times 35.$

Q. If a number of horses eat $35 worth of oats in 5 days, will it cost more or less to feed the same number of horses 8 days? *Ans.* More, $\frac{8}{5}$ as much. Write $\frac{8}{5}$ at the left of the other numbers. $\frac{8}{5} \times \frac{60}{37} \times 35.$

6. If 30 masons in 15 days can build a wall 387.5 ft. long, what is the length of a wall that can be built by 48 masons in 20 days?

What is asked for in this problem? What is the number to be multiplied by the fractions to be found?

If 30 masons can build a wall 387.5 ft. long in a given time, will 48 masons build a greater or less length of wall in the same time? What is the resulting fraction?

If a number of masons can build a wall 387.5 ft. long in 15 days, can they build a greater or less length of wall in 20 days? What is the resulting fraction?

7. If it takes 60 men 12 days to make a pavement 160 feet long and 40 feet wide, how long will it take 54 men to make a similar pavement 180 ft. long 48 ft. wide?

8. If 180 men in 6 days of $6\frac{2}{3}$ hours each can dig a ditch 600 yd. long, 3 yd. wide, 2 yd. deep, in how many days can 100 men working $5\frac{1}{3}$ hours a day dig a ditch 540 yd. long 4 yd. wide, and 3 yd. deep?

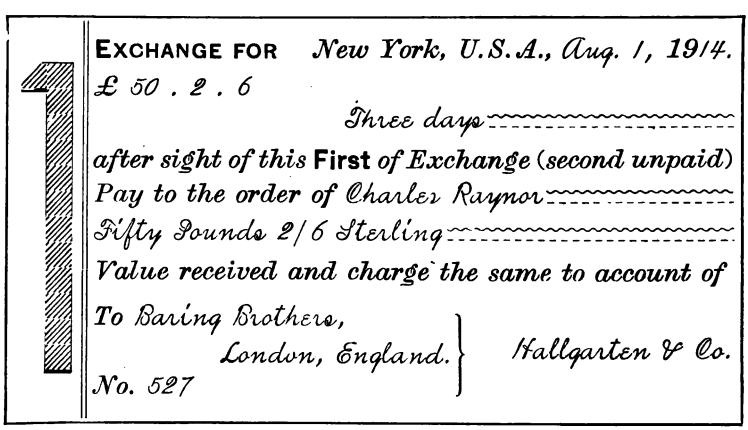
9. 36 men are engaged to do a piece of work in 48 days. After 12 days, 12 men stop. In how many days can the rest finish the work?

FOREIGN EXCHANGE

Exchange between places in different countries is called **foreign exchange.**

Foreign exchanges are usually transacted by *international money orders, letters of credit, travelers' checks* (see p. 374), and *bills of exchange.*

A **foreign bill of exchange** is a draft of a bank in one country upon a bank in another. It is payable in the money of the country on which it is drawn.



To guard against loss, foreign bills of exchange are usually issued in duplicate. The duplicate reads "Second of Exchange (first unpaid)" and the number 2 appears instead of 1 in the left margin.

Problems in foreign exchange are solved in the same manner as in domestic exchange, except that the currency of one country must be reduced to that of another.

Exchange on England is quoted by giving the cost of a bill for £1. Thus, 4.86 means that a bill of exchange for £1 costs $4.86; German exchange is quoted by giving the number of cents that 4 marks cost. Thus, a quotation of 95⅛ means that 4 marks cost 95⅛¢; French exchange is quoted by giving the number of francs that can be bought for $1. Thus, 5.17 means that 5.17 francs can be bought for $1.

PROBLEMS

WRITTEN

1. What is the cost of a draft on London for £380 15*s*., exchange at $4.8665?

SOLUTION. £380 15*s*. = £380.75; 380.75 × $4.8665 = $1852.92, cost of draft.

2. Find the cost of a bill of exchange on Berlin for 1750 marks, exchange being at 96¼.

SOLUTION. Since 4 marks cost $.96¼, 1750 marks will cost (1750 ÷ 4) × $.96¼, or $421.09, cost of draft.

3. Find the cost of a bill of exchange on Paris for 2060 francs when exchange is quoted at 5.15.

SOLUTION. Since 5.15 francs cost $1, 2060 francs cost (2060 ÷ 5.15) × $1, or $400, cost of draft.

Find, to the nearest cent, the cost of exchange for:

4. £200 @ $4.87.
5. 5000 francs @ 5.17.
6. 10,000 M. @ 95⅛.
7. £500 12*s*. @ 4.88.

8. How much must a merchant in Canton pay for a draft of $1134, if 1 tael costs $.642?

TABLES FOR REFERENCE

Measure of Length

12 inches = 1 foot
3 feet = 1 yard
16½ feet = 1 rod
320 rods = 1 mile (statute)

A *nautical mile* (*knot*) = 6080.27 ft., or about 1.15 mi.

A *hand*, used in measuring the height of horses, = 4 in.; a *fathom*, used in measuring the depth of water, = 6 ft.; a *furlong* = ⅛ mi.

Measure of Surface

144 square inches = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod
160 square rods = 1 acre

An acre of land in the form of a square is very nearly 209 ft. on a side.

A *square* of roofing or flooring equals 100 sq. ft.

Measure of Volume

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
128 cubic feet = 1 cord

Surveyors' Linear Measure

7.92 inches = 1 link
25 links = 1 rod
100 links = 1 chain
80 chains = 1 mile

This chain, called *Gunter's chain*, is 4 rd., or 66 ft., long.

Surveyors' Square Measure

16 square rods = 1 square chain
10 square chains = 1 acre
640 acres = 1 square mile

Liquid Measure

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
31½ gallons = 1 barrel

1 gal. = 231 cu. in.; 1 cu. ft. = about 7½ gal. A gallon of water weighs about 8⅓ lb.; a cubic foot of water weighs about 62½ lb., or 1000 oz.

Dry Measure

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

1 bu. = 2150.42 cu. in., or about 1¼ cu. ft. The heaped bu. = 2747.71 cu. in., or nearly 1⅝ cu. ft. 1 qt. or ½ bu. = 67.2 cu. in.

Avoirdupois Weight

16 ounces = 1 pound
7000 grains = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 1 ton
2240 pounds = 1 *long* ton
20 cwt. = 1 ton
1 av. oz. = 437½ grains.

* 32 pounds = 1 bu. of oats
48 pounds = 1 bu. of barley
* 56 pounds = 1 bu. of shelled corn, or rye; also 1 firkin of butter
* 60 pounds = 1 bu. of wheat, potatoes, or clover seed
70 pounds = 1 bu. of corn on cob
80 pounds = 1 bu. of coal
196 pounds = 1 bbl. of flour
200 pounds = 1 bbl. of beef or pork

* In most states.

Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound
5760 grains = 1 pound

The unit generally used for weighing diamonds, gems, etc., is the *carat*, which is about 3.2 Troy grains. It is used also to express the fineness of gold. 18 carats fine means $\frac{18}{24}$ pure gold and $\frac{6}{24}$ baser metal.

The Troy pound = 5760 grains
The Troy ounce = 480 grains

Apothecaries' Weight

20 grains = 1 scruple — sc. or ℈
3 scruples = 1 dram — dr. or ʒ
8 drams = 1 ounce — oz. or ℥
12 ounces = 1 lb. or ℔

This table is used only in filling medical prescriptions.

Apothecaries' Liquid Measure

60 minims (m) = 1 fluid dram (f ʒ)
8 fluid drams = 1 fluid ounce (f ℥)
16 fluid ounces = 1 pint (O.)
8 pints = 1 gal. (Cong.)

This table is used only in filling medical prescriptions.

Counting Table

12 things = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross
20 things = 1 score

Stationers' Table

24 sheets = 1 quire
20 quires = 1 ream

Paper is frequently sold by the pad or bulk of 100, 500, or 1000 sheets, or by the pound.

Time Measure

60 seconds	= 1 minute
60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
365 days	= 1 year
366 days	= 1 leap year
10 years	= 1 *decade*
100 years	= 1 *century*

Thirty days have September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four.

Measure of Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 4 right angles or 1 circumference
90° of angle = 1 *right angle;* 90° of arc = 1 *quadrant.*

United States Money

10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

English Money

4 farthings = 1 penny
12 pence = 1 shilling
20 shillings = 1 pound
1 pound (£) = $4.8665

German Money

1 mark (M.) = $.238

French Money

1 franc (fr.) = $.193

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ANSWERS

TO

HARVEY'S ESSENTIALS OF ARITHMETIC

SECOND BOOK — PART II

Page ix. — **1.** *a.* 180,157; *b.* 315,904; *c.* 370,783; *d.* $2746.25; *e.* $2847.21. **2.** *a.* 14,536; *b.* 12,686; *c.* 13,293; *d.* 18,814; *e.* 964. **3.** *a.* $216.79; *b.* $345.11; *c.* $1.05; *d.* $837.22; *e.* $779.07. **4.** *a.* 7,051,789; *b.* 12,856,050; *c.* 12,126,312; *d.* 23,003,100; *e.* 57,739,110. **5.** *a.* $196,522.95; *b.* $145,611.84; *c.* $612,-928.50; *d.* $668,675.25; *e.* $5,462,940.40. **6.** *a.* 729; *b.* 353; *c.* $8.44. **7.** *a,* 867; *b.* 404; *c.* $69.60. **8.** *a.* 628; *b.* 2080; *c.* $2.08. **9.** *a.* 2270; *b.* 208; *c.* $39.60. **10.** *a.* 994; *b.* 213; *c.* $5.16.

Page x. — **11.** $\frac{2}{3}$; $\frac{3}{8}$; $\frac{1}{4}$; $\frac{8}{15}$; $\frac{3}{5}$; $\frac{5}{6}$; $\frac{3}{4}$; $\frac{2}{3}$. **12.** $\frac{5}{12}$; $\frac{3}{4}$; $\frac{3}{10}$; $\frac{4}{5}$; $\frac{7}{8}$; $\frac{7}{8}$; $\frac{3}{5}$; $\frac{13}{14}$. **13.** $\frac{28}{4}$; $\frac{50}{8}$; $\frac{75}{2}$; $\frac{125}{2}$; $\frac{87}{8}$; $\frac{47}{3}$; $\frac{99}{8}$; $\frac{78}{5}$. **14.** *a.* $1\frac{1}{2}$; $1\frac{5}{8}$; $\frac{11}{15}$. *b.* $1\frac{11}{14}$; $1\frac{27}{40}$; $1\frac{11}{20}$. *c.* $\frac{11}{16}$; $\frac{1}{2}$; $\frac{11}{24}$. *d.* $\frac{3}{16}$; $\frac{1}{15}$; $\frac{1}{20}$. **15.** $51\frac{7}{4}$, $18\frac{3}{8}$; $110\frac{1}{4}$, $38\frac{5}{8}$; $139\frac{7}{8}$, $26\frac{3}{8}$; $59\frac{1}{4}$, $25\frac{4}{5}$; $115\frac{1}{2}$, $43\frac{9}{10}$. **16.** $133\frac{1}{2}$, $64\frac{1}{8}$; $98\frac{5}{8}$, $14\frac{1}{6}$; $140\frac{5}{8}$, $47\frac{1}{8}$; $119\frac{8}{10}$, $1\frac{7}{10}$; $89\frac{1}{5}$, $11\frac{1}{5}$. **17.** $\frac{5}{8}$; $7\frac{3}{4}$; $\frac{7}{10}$; $1\frac{1}{10}$. **18.** 10; $83\frac{17}{24}$; $5\frac{7}{8}$; $5\frac{3}{11}$. **19.** $\frac{7}{15}$; $115\frac{5}{13}$; $\frac{5}{8}$; $6\frac{3}{13}$. **20.** 551.3724, 160.6376; 437.574, 404.426; 88.4032, 85.7108; 934.605, 108.195. **21.** 839.4449, 208.5715; 302.715, 115.365; 1599.035, 314.965; 1151.995, 598.105. **22.** 1390.8173; 740.289; 1687.4382; 2086.6.

Page xi. — **23.** *a.* 27.75; *b.* 1.7879004; *c.* 126. **24.** *a.* .359784. *b.* 5.035304; *c.* 6.9. **25.** *a.* .028899; *b.* .401268; *c.* 12.6. **26.** *a.* .0288796; *b.* 616; *c.* 670. **27.** *a.* 127.635; *b.* 17; *c.* 3012.

1. $8,000,000. **2.** 16 stations. **3.** $.12\frac{1}{2}$; $.00\frac{1}{2}$ a pound cheaper. **4.** $276. **5.** 1,454,000 birds. **6.** 8,724,000,000 insects. **7.** 81,000 ants.

Page xii. — **8.** 210 locusts. **9.** 235,368,000 locusts. **10.** $2521.80. **11.** $8\frac{1}{3}$ da. **12.** 289.8 lb.

Page xiii. — **14.** $\frac{1}{4}$ c. butter, $\frac{1}{4}$ c. sugar, $\frac{1}{4}$ c. molasses, $1\frac{1}{4}$ c. flour, $1\frac{1}{4}$ level tsp. soda, $\frac{1}{2}$ level tsp. cinnamon, $\frac{1}{2}$ level tsp. ginger, $\frac{1}{2}$ level tsp. cloves, $\frac{1}{2}$ c. boiling water, 1 egg, $\frac{1}{2}$ tsp. salt; $\frac{1}{8}$ c. butter, $\frac{1}{8}$ c. sugar, $\frac{1}{8}$ c. molasses, $\frac{5}{8}$ c. flour, $\frac{5}{8}$ level tsp. soda, $\frac{1}{4}$ level tsp. cinnamon, $\frac{1}{4}$ level tsp. ginger, $\frac{1}{4}$ level tsp. cloves, $\frac{1}{4}$ c. boiling water, $\frac{1}{2}$ egg, $\frac{1}{2}$ tsp. salt. **16.** 3 c. flour. **17.** $\$.31\frac{7}{12}$.

Page 251. — **14.** 606 hr. **15.** 150 min. **16.** 920 sec. **17.** 365 da. **18.** 174 da. **19.** 8120 min. **20.** 45,318 sec. **21.** 248 oz. **22.** 120 oz. **23.** 16,750 lb. **24.** 278 hr. **25.** 8500 lb.

Page 252. — **26.** 222 in. **27.** 62 ft. **28.** 90 yd. **29.** 159 yd. **30.** 51 yd. 1 ft. 6 in. **31.** 64 yd. 1 ft. 6 in. **32.** 1763 yd. **33.** 206 ft. **34.** 408 ft. **35.** $125\frac{1}{2}$ ft. **36.** $453\frac{1}{2}$ ft. **37.** 155 ft. 6 in.

Page 253. — **18.** 7 gal. 1 pt. **19.** 6 yd. 1 ft. 7 in. **20.** 1 da. 17 hr. 15 min. **21.** 17 cu. yd. 16 cu. ft. **22.** 7 bu. 2 pk.

Page 254. — **23.** 10 rd. 1 yd. **24.** 3 mi. 280 rd. **25.** 25 cwt. 60 lb. **26.** 4 lb. 11 oz. **27.** $12\frac{5}{32}$ bu. **28.** $19.44. **29.** $17.20. **30.** $49.50. **31.** $450, **32.** $7.20. **33.** $4.41. **34.** $1.89. **35.** 10,800 shingles.

Page 255. — **6.** 21 hr. **7.** 96 cu. ft. **8.** 3 gal. 3 qt. 1½ pt. **9.** 2 ft. 7½ in. **10.** 3 qt. **12.** 1.68 hr.; 3.6 hr.

Page 256. — **13.** .4 qt.; 3.6 qt. **16.** .03 wk. **17.** .09625 mi. **20.** $\frac{9}{32}$. **21.** $\frac{5}{10}$; .9. **22.** *a.* $\frac{7}{16}$ lb.; .4375 lb. *b.* ¾ mi.; .75 mi. *c.* $\frac{19}{20}$ T.; .95 T. *d.* ⅔ yd.; .66⅔ yd. *e.* ⅔ hr.; .66⅔ hr. *f.* $\frac{5}{12}$ ft.; .41⅔ ft. **23.** *a.* ⅝ mi.; .375 mi. *b.* ⅝ mi.; .6 mi. *c.* ⅘ T.; .8 T. *d.* ½ da.; .33⅓ da. *e.* ⅔ min.; .66⅔ min. *f.* ⅜ T.; .375 T.

Page 258. — **5.** 6 mi. 68 rd. 3 yd. 1 ft. 6 in. **6.** 7 mi. 221 rd. 4 ft. **7.** 51 bu. 2 pk. 2 qt. **8.** 16 T. 253 lb. **9.** 49 hr. 27 min. **10.** 16 sq. yd. 2 sq. ft. **11.** 43 cu. ft. 1670 cu. in. **12.** 4 hr. 25 min. **13.** 240 rd. **14.** 4 doz. 3 units.

Page 259. — **15.** 5 bu. 4 qt. **16.** 5 gal. 3 qt. **17.** 6 yd. 2 ft. 10. in. **2.** 69 yr. 2 mo. 15 da. **4.** 1 yr. 9 mo. 27 da. **5.** 7 mo. 26 da. **6.** 10 mo. 29 da. **7.** 1 yr. 7 mo. 27 da. **8.** 5 yr. 8 mo. 28 da. **9.** 7 yr. 7 mo. 23 da. **10.** 7 yr. 3 mo. 27 da.

Page 260. — **6.** 7 rd. 3 yd. 2 ft. 6 in. **7.** 26 A. 123½ sq. rd.

Page 261. — **8.** 3 mi. 150 rd. **9.** 13 pieces; 1½ in. left. **10.** 1 lb. 9 oz. **12.** 8 pk. 2 qt. **13.** 13 bu. 3 pk. **14.** 27 lb. 12 oz. **15.** 11 T. 500 lb. **16.** 136 yd. **17.** 16 yd. 1 ft. 6 in. **18.** 22 yd. **19.** 50 rd. 5 yd. **20.** 13 gal. 1 pt. **21.** 28 gal. 2 qt. **22.** 20 mi. 160 rd. **23.** 15 sq. yd. 1 sq. ft. **24.** 9 cu. yd. 13 cu. ft. **25.** 23 hr. 20 min. **26.** 6 mi. 24 rd. 4 yd. **27.** 11 gal. 2 qt. 1 pt. **28.** 24 bu. 3 pk. 6 qt. **29.** 36 rd. 4 yd. 2 ft. **30.** 2 qt. 1 pt. **31.** 1 hr. 15 min. **32.** 230 rd. **33.** 1 ft. 4 in. **34.** 1 yd. 1 ft. **35.** 1400 lb. **36.** 14 cu. ft. **37.** 3 yd. **38.** 4. **39.** 3.

Page 262. — **1.** 7 T. 1687 lb. **2.** $13.02. **3.** $929.81. **4.** $4.75. **5.** 14,400 shingles. **6.** 58 bunches; $50.75. **7.** 46 A. 100 sq. rd. **8.** 21⅗ T. **9.** $40 gain.

Page 263. — **10.** $25; $20; $17.50. **11.** $54.03. **12.** $126\frac{23}{460}$ T. **13.** $60.30. **14.** $178.20. **15.** $81. **16.** 90 gal. **17.** $39.90. **18.** 7 mi. 220 rd.

Page 264. — **19.** 30 mi. an hr. **20.** 328.875 T. **21.** 24 steps. **22.** 6204 steps. **23.** 720 times. **24.** 8910 times. **25.** 8 strips; 10 strips. **26.** 8 strips; 9 strips. **27.** 6 in.; one extra strip 22′ long. **28.** 288 sq. rd.

Page 265. — **29.** 1⅘ A. **30.** $1619.20. **31.** $30. **32.** 302⅝ cu. in. **33.** $41.25. **34.** 12 bbl. **36.** 91 hr. **37.** 6760 ft. **38.** 2 mi. 2880 ft.

Page 266. — **39.** $59.26. **40.** 43$\frac{31}{32}$ A. **41.** 6¼ A. **42.** 40 ft.; 24 ft. **43.** 192 sq. ft.; 128 sq. ft. **44.** $576. **46.** 1296 sq. in. **47.** 29⅓ bd. ft. required; 11⅓ bd. ft. wasted.

Page 267. — **48.** 224 sq. ft.; 104 sq. ft. **49.** $1975. **50.** 64 sq. in.; 48 sq. in.; 96 sq. in.; 64 sq. in. more. **51.** 28″ × 4″. **52.** 208 sq. in. **53.** 126 sq. yd. **55.** $9000; $2250.

Page 268. — **56.** 864 sq. ft. **57.** 30 sq. yd.; 40 yd.; crosswise. **58.** $2.40; $12.91. **59.** 288 tiles. **60.** 128 sq. ft. **61.** 70 bd. ft. **62.** 216 bd. ft. **63.** $13.72. **64.** $9.10.

Page 269. — **65.** $22\frac{1}{2}$ M shingles; \$90. **68.** \$82.63, strips running lengthwise. **69.** $281\frac{7}{18}$ sq. ft. **70.** \$20. **71.** $10' \times 10'$; 100 sq. ft.; 50'. **72.** $15{,}697\frac{11}{17}$ times. **73.** 14 gal. 1 qt. **74.** 16 baskets.

Page 270. — **75.** 11 ties; 13 ties. **76.** \$1056. **77.** 37 bbl., 1 partly filled; 4 pk. **78.** 9 da. **79.** 34 mi. per hr. **80.** \$9.19. **81.** \$18.74. **82.** 35 bu. 3 pk. 4 qt. **83.** 1498 bu. 3 pk. 4 qt. **84.** .625 bu. **85.** \$.40; \$.10. **86.** 6 gross. **87.** \$3.70; \$9.25.

Page 271. — **88.** Approximate, 360 gal., $38\frac{2}{5}$ bu.; exact, $359\frac{5}{77}$ gal.; $38\frac{4}{7}$ bu. **89.** 96 sq. in. **90.** $47\frac{11}{17}$ cu. yd. **91.** 192 bu. of oats; 3 T. 144 lb. **92.** Approximate, 6000 gal.; exact, $5984\frac{2}{7}$ gal. **93.** 216 cu. ft. **94.** 144 bd. ft. **95.** 260 bottles. **96.** $\frac{14}{33}$ mi. **97.** 14,400 shingles.

Page 272. — **98.** 9 shingles.

Page 277. — **18.** $\frac{3}{4}$. **19.** $\frac{25}{6}$ or $4\frac{1}{6}$. **20.** *a.* $\frac{3}{4}$; *b.* $\frac{8}{5}$ or $1\frac{3}{5}$; *c.* $\frac{12}{35}$; *d.* $\frac{13}{14}$. *e.* $\frac{17}{24}$; *f.* $\frac{5}{16}$. **21.** *a.* $\frac{5}{11}$; *b.* $\frac{3}{4}$; *c.* $\frac{6}{25}$; *d.* $\frac{9}{20}$; *e.* $\frac{3}{13}$; *f.* $\frac{13}{3}$ or $4\frac{1}{3}$. **22.** *a.* $\frac{3}{8}$; *b.* 2; *c.* $\frac{3}{2}$ or $1\frac{1}{2}$; *d.* $\frac{24}{55}$; *e.* $\frac{6}{17}$; *f.* $\frac{25}{54}$. **23.** *a.* $\frac{1}{2}$; *b.* $\frac{1}{2}$; *c.* 3; *d.* $\frac{2}{3}$; *e.* $\frac{5}{7}$; *f.* $\frac{2}{3}$. **24.** $\frac{7}{10}$. **25.** $\frac{11}{21}$ or $1\frac{10}{21}$. **26.** $\frac{1}{3}$. **27.** $\frac{132}{33}$ or 4.

Page 278. — **5.** 16. **6.** 6. **7.** 14. **8.** 8. **9.** 10. **10.** 5. **11.** 5. **12.** 16. **13.** 4. **14.** 20. **15.** 5. **16.** 12.

Page 280. — **5.** 144 bu. **6.** 10 da. **7.** 8 men. **8.** 34 lb.

Page 281. — **9.** $187\frac{1}{2}$ bu. **10.** 100 da. **11.** 7 hr. **12.** 8 hr. **13.** 40 additional workmen. **14.** 5¢. **15.** 6 da. **16.** 24 farm hands.

Page 282. — **2.** 9; 15. **3.** 15; 21. **4.** 9; 6. **5.** James, 8 pieces; John, 12 pieces. **6.** 3 : 4; 4 : 3. **7.** James, 15 pieces; John, 25 pieces. **8.** 12 mi.; 18 mi.; 6 hr. **9.** 4 ft.; 6 ft.; 8 ft.

Page 288. — **20.** 1.96; 2.73; 17.57; 12.25; 42; .35; 28; 21; 70. **21.** 4.32; 5.88; 10.44; 14.4; 16.8; 19.2; 21.6; 28.8; 36. **22.** 1.08; 2.16; 2.736; 1.638; .33; .276; .432; 54; 30. **23.** .216; .28; .2224; 1.248; .272; .44; .52; 60; 64. **24.** 4.5; 1.44; 8.2; .86; .162; .195; .234; 28.2; 26.1. **25.** 1.68; .84; 1.26; 4.375; .945; .1225; .2345; .189; 28; 24.5. **26.** 1.875; 3; 1.5; 2.625; 31.95; 60; 48; 39.6; 31.2. **27.** \$12. **28.** \$3. **29.** 18 lb. **30.** 6 oz. **31.** $10\frac{1}{2}$ da. **32.** \$10.80. **33.** 9 mo. **34.** \$2. **35.** 12 eggs. **36.** \$8. **37.** 12 cents. **38.** 4. **39.** \$1.50. **40.** 5. **41.** 72.9. **42.** .65. **43.** \$10. **44.** 8.75 acres. **45.** 18 eggs. **46.** \$1100. **47.** $\$33\frac{1}{3}$. **48.** 3; 7; 10.375; 16; 31.25; 37.5; 75; 112.5. **49.** 4; 7; $8\frac{1}{2}$; $11\frac{1}{2}$; $35\frac{1}{2}$; 50; 75; 100. **50.** 18; 42; 43.2; 57.6; 32.4; 120; 180; 360. **51.** .49; 31.5; 33.25; 73.5; 220.5; 700; 1400; 1575.

Page 289. — **52.** 25%: *a.* \$50; *b.* 3905; *c.* 38.5 mi.; *d.* \$100.20; *e.* 4000. 50%: *a.* \$100; *b.* 7810; *c.* 77 mi.; *d.* \$200.40; *e.* 8000. $37\frac{1}{2}$%: *a.* \$75; *b.* 5857.5; *c.* 57.75 mi.; *d.* \$150.30; *e.* 6000. $112\frac{1}{2}$%: *a.* \$225; *b.* 17572.5; *c.* 173.25 mi.; *d.* \$450.90; *e.* 18,000. 5%: *a.* \$10; *b.* 781; *c.* 7.7 mi.; *d.* \$20.04; *e.* 800. **53.** 25%: *a.* \$79; *b.* 4472.5; *c.* 66 yd.; *d.* \$79.10; *e.* 4500. 50%: *a.* \$158; *b.* 8945; *c.* 132 yd.; *d.* \$158.20; *e.* 9000. $37\frac{1}{2}$%: *a.* \$118.50; *b.* 6708.75; *c.* 99 yd.; *d.* \$118.65; *e.* 6750. $112\frac{1}{2}$%: *a.* \$355.50; *b.* 20,126.25; *c.* 297 yd.; *d.* \$355.95; *e.* 20,250. 5%: *a.* \$15.80; *b.* 894.5; *c.* 13.2 yd.; *d.* \$15.82; *e.* 900. **54.** 25%: *a.* \$620; *b.* 11,000; *c.* 86 gal.; *d.* \$2370.12; *e.* 6000. 50%: *a.* \$1240; *b.* 22,000; *c.* 172 gal.; *d.* \$4740.24; *e.* 12,000. $37\frac{1}{2}$%: *a.* \$930; *b.* 16,500;

c. 129 gal.; *d.* $3555.18; *e.* 9000. 112½%: *a.* $2790; *b.* 49,500; *c.* 387 gal. *d.* $10,665.54; *e.* 27,000. 5%: *a.* $124; *b.* 2200; *c.* 17.2 gal.; *d.* $474.02; *e.* 1200. **55.** 2%: *a.* 32.8; *b.* 315.6; *c.* $2.51; *d.* $46.82; *e.* $141.61. 3½%: *a.* 57.4; *b.* 552.3; *c.* $4.39; *d.* $81.93; *e.* $247.81. 5¼%: *a.* 94.3; *b.* 907.35; *c.* $7.22; *d.* $134.60; *e.* $407.11. 6%: *a.* 98.4; *b.* 946.8; *c.* $7.53; *d.* $140.45; *e.* $424.82. **56.** 2%: *a.* 59.4; *b.* 400; *c.* $2.33; *d.* $70.01; *e.* $40.81. 3½%: *a.* 103.95; *b.* 700; *c.* $4.09; *d.* $122.51; *e.* $71.42. 5¼%: *a.* 170.775; *b.* 1150; *c.* $6.72; *d.* $201.27; *e.* $117.33. 6%: *a.* 178.2; *b.* 1200; *c.* $7.01; *d.* $210.02; *e.* $122.43. **57.** 2%: *a.* 166.82; *b.* 1000; *c.* $4.81; *d.* $120.01; *e.* $197.10. 3½%: *a.* 291.935; *b.* 1750; *c.* $8.42; *d.* $210.02; *e.* $344.92. 5¼%: *a.* 479.61; *b.* 2875; *c.* $13.83; *d.* $345.04; *e.* $566.65. 6%: *a.* 500.46; *b.* 3000; *c.* $14.43; *d.* $360.04; *e.* $591.29. **58.** 37½%: *a.* 761.25; *b.* 6937.5; *c.* $240.19; *d.* $2437.69; *e.* $3750. 137½%: *a.* 2791.25; *b.* 25,437.5; *c.* $880.69; *d.* 8938.19; *e.* $13,750. 133⅓%: *a.* 2706⅔; *b.* 24,666⅔; *c.* $854; *d.* $8667.33; *e.* $13,333.33. ¼%: *a.* 5.075; *b.* 46.25; *c.* $1.60; *d.* $16.25; *e.* $25. .05%: *a.* 1.015; *b.* 9.25; *c.* $.32; *d.* $3.25; *e.* $5. **59.** 37½%: *a.* 1143 75; *b.* 6675; *c.* $273.84; *d.* $3469.03; *e.* $9375. 137½%: *a.* 4193.75; *b.* 24.475; *c.* $1004.09; *d.* $12,719.78; *e.* $34,375. 133⅓%: *a.* 4066⅔; *b.* 23,733⅓; *c.* $973.67; *d.* $12,334.33; *e.* $33,333.33. ¼%: *a.* 7.625; *b.* 44.5; *c.* $1.83; *d.* $23.13; *e.* $62.50. .05%: *a.* 1.525; *b.* 8.9; *c.* $.37; *d.* $4.63; *e.* $12.50. **60.** 37½%: *a.* 2276.25; *b.* 7275; *c.* $309.44; *d.* $3017.10; *e.* $30,000. 137½%: *a.* 8346.25; *b.* 26,675; *c.* $1134.60; *d.* $11,062.70; *e.* $110,000. 133⅓%: *a.* 8093⅓; *b.* 25,866⅔; *c.* $1100 21; *d.* $10,727.47; *e.* $106,666.67. ¼%: *a.* 15.175; *b.* 48.5; *c.* $2.06; *d.* $20.11; *e.* $200. .05%: *a.* 3.035; *b.* 9.7; *c.* $.41; *d.* $4.02; *e.* $40.

Page 291.—**19.** 850 A.; 85 A.; 340 A. remained. **20.** 66⅔ lb. **21.** $22,255.82.

Page 292.—**22.** 1.741 lb. **23.** $6.08. **24.** $77.01; $3.08. **25.** Protein, 24 lb.; carbohydrates, 280 lb.; fat, 14 lb. **26.** Shallow setting, 8 lb.; deep setting, 2 lb.; separator, .5 lb. **27.** Shallow setting, 960 lb., $288; deep setting, 240 lb., $72; separator, 60 lb., $18.

Page 293.—**28.** $3.36. **29.** Fat, 2490 lb.; water, 390 lb.; casein, 30 lb.; salt, 90 lb. **30.** 569.4129 lb. **31.** 243.01632 lb.; 79,223.3203 lb. **32.** 24.7152 lb.; $3.37. **33.** 23.9856 lb.; $2.78. **34.** 24.6129 lb.; $4.83. **35.** 24.6995 lb.; $3.64.

Page 294.—**36.** 25.856 lb.; $3.51. **37.** 24.628 lb.; $7.17. **38.** 20.6178 lb.; $6. **39.** 11.55 lb.; $3.62. **40.** 2.3436 lb.; $.81. **42.** 31.4146 lb.; $7.58. **43.** 32.0901 lb.; $7.21. **44.** 34.068 lb.; $8.91. **45.** 156,317.1604 lb. **46.** 49,090.4815 lb. **47.** 62,985.5492 lb. **48.** 93,830.49 lb. **49.** 3.9%; 2.14 lb. **50.** The cow in Ex. 49. **51.** $32.20. **52.** 2.5 lb. **53.** 1.92 lb. **54.** 1.125 lb. **55.** 3.2 lb.

Page 295.—**56.** 3.3 lb. **57.** 1.575 lb. **58.** Cream is $87.50 more profitable.

	Nitrogen	Phosphoric Acid	Potash	Value
59. Horse,	11.6 lb.	5.6 lb.	10.6 lb.	$2.55
60. Cow,	8.8 lb.	3.2 lb.	8 lb.	$1.88
61. Pig,	9 lb.	3.8 lb.	12 lb.	$2.14
62. Sheep,	16.6 lb.	4.6 lb.	13.4 lb.	$3.39
63. Poultry,	32 lb.	30 lb.	16 lb.	$7.10

64. $166\frac{2}{3}$ lb. each of cornmeal, ground oats, and bran; 1750 lb. corn silage; 750 lb. red clover.

	Dry Matter	Nitrogen	Phosphoric Acid	Potash
65. Cornmeal,	$145.1\frac{2}{3}$ lb.	$2.6\frac{1}{6}$ lb.	1.05 lb.	$\frac{2}{3}$ lb.
66. Oats,	$148.1\frac{2}{3}$ lb.	3.1 lb.	$1.28\frac{1}{3}$ lb.	$.98\frac{1}{3}$ lb.
67. Bran,	$147.1\frac{2}{3}$ lb.	4.45 lb.	$4.81\frac{2}{3}$ lb.	$2.68\frac{1}{3}$ lb.
68. Silage,	385 lb.	4.9 lb.	1.925 lb.	6.475 lb.
69. Clover,	665.25 lb.	15.525 lb.	2.85 lb.	16.5 lb.

Page 296. — **70.** 8937.$6\frac{1}{4}$ lb. nitrogen; 3482.1 lb. phosphoric acid; 7974.$03\frac{1}{4}$ lb. potash. **71.** 11,865 lb. nitrogen; 549.5 lb. phosphoric acid; 1697.5 lb. potash.

Page 297. — **16.** $37\frac{1}{2}\%$. **17.** $17\frac{43}{121}\%$. **18.** $12\frac{1}{2}\%$. **19.** $11\frac{1}{9}\%$. **20.** $37\frac{1}{2}\%$. **21.** 66%. **22.** $66\frac{2}{3}\%$. **23.** 60%. **24.** 120%. **25.** 300%. **26.** $214\frac{2}{7}\%$. **27.** 120%.

Page 298. — **11.** $4\frac{8}{23}\%$. **12.** $7\frac{14}{23}\%$. **13.** $11\frac{22}{23}\%$. **14.** $6\frac{12}{23}\%$. **15.** $5\frac{10}{23}\%$. **16.** $30\frac{10}{23}\%$. **17.** $14\frac{8}{23}\%$. **18.** $19\frac{18}{23}\%$.

	Soil	Nitrogen	Phosphoric Acid	Potash
19.	1	.2185%	.0585%	1.612%
20.	2	.242%	.06%	1.647%
21.	3	.2145%	.0595%	1.767%
22.	4	.2455%	.061%	1.648%
23.	5	.2525%	.0595%	1.8125%
24.	6	.3375%	.0705%	2.251%

Page 299. — **25.** 3.86%. **26.** 4%. **27.** 5.14%. **28.** 6.03%. **29–32.** (32) 63.34%. (33) 52.25%. (34) 127.77%. (35) 73.23%.

Page 301. — **18.** 400; 700; 280; 200; 560. **19.** 200; 600; 300; 180; 900. **20.** 400; 200; 600; 300; 800.

Page 302. — **21.** 100; 300; 200; 600. **22.** 100; 300; 1500; 500. **23.** 600; 1000; 1200; 400. **24.** 400; 200; 1000; 800. **25.** 300; 1100; 330. **26.** 400; 800; 500; 200. **27.** 700; 300; 210; 168. **28.** 900; 500; 450; 135. **30.** 271$\frac{1}{3}$. **31.** 1600; 3200; 2560. **32.** 2083$\frac{1}{3}$; 1666$\frac{2}{3}$; 1388$\frac{8}{9}$. **33.** 312.5; 1250; 125. **34.** 54$\frac{1}{8}$; 32.5; 26. **35.** 585; 292.5; 234. **36.** \$55.50; \$27.75. **37.** \$148.32; \$16.48; \$30.90; \$20.60; \$15.45; \$32.96. **38.** \$36.45; \$72.90; \$38.88; \$145.80; \$194.40.

Page 303. — **39.** $3\frac{1}{4}$; 3; $2\frac{1}{2}$; 2; $7\frac{1}{2}$; 6.

Page 304. — **8.** 400. **9.** 720. **10.** \$800. **11.** \$3000. **12.** \$2000. **13.** \$1500. **14.** \$2000. **15.** \$1600. **16.** \$1000. **17.** \$1600. **18.** \$1600. **19.** \$2000. **20.** \$3000. **21.** \$3000. **22.** \$3000. **23.** \$2000. **24.** \$2000. **25.** 2000. **26.** 3000. **27.** 2000. **28.** 500. **29.** 800. **30.** \$4000. **31.** \$9000. **32.** \$8000. **33.** \$8000. **34.** \$2250. **35.** \$8800. **36.** \$4000. **37.** \$2000. **38.** \$2000. **39.** \$10,000. **40.** \$2000. **41.** \$10,000. **42.** \$10,000. **43.** \$3000. **44.** \$10,000. **45.** 10,000. **46.** 10,000. **47.** 10,000.

Page 307. — **11.** \$3.30. **12.** \$31. **13.** \$18.70. **14.** 6%. **15.** $66\frac{2}{3}\%$. **16.** \$231.25.

Page 308. — **17.** $41\frac{2}{3}\%$. **18.** $17\frac{3}{16}\%$. **20.** \$7.50 gain. **21.** 50%. **22.** $66\frac{2}{3}\%$. **23.** $33\frac{1}{3}\%$ gain; 20% gain. **24.** 25%.

Page 309. — **25.** 100 %. **26.** 15 % loss; 17$\frac{11}{17}$ % gain. **27.** 4.5 % gain. **28.** $60. **29.** 130 bu. **30.** $.80; 32 %. **31.** 25 %.

Page 311. — **11.** $36. **12.** $75. **13.** $120. **14.** $135. **15.** $475. **16.** $658. **17.** $420. **18.** $450.

Page 312. — **1.** $12; $4.80; $43.20. **2.** $54. **3.** $152. **4.** Single discount of 53 %. **5.** 46 %, or $55.20. **6.** 32 %. **7.** $7.47. **8.** $427.50. **9.** $2475. **10.** $655.50; $690.

Page 314. — **5.** $28.25; $536.75. **6.** $300. **7.** 2$\frac{1}{2}$ %. **8.** $660. **9.** $300; $285. **10.** $4.50.

Page 315. — **11.** $3220. **12.** $17.27. **13.** $1.025. **14.** Yearly salary of $1800; $360. **15.** $3756.25. **16.** $365.75. **17.** $47,632.50. **18.** $215; $140.

Page 317. — **3.** $50.40. **4.** $3.30. **5.** $36. **6.** $189.88. **7.** $280. **8.** $8400. **9.** 80 ¢ a hundred; $\frac{1}{8}$ %. **10.** $34,775. **11.** $23,500; $3571.43.

Page 319. — **4.** $210 more profit to rent. **5.** $.011$\frac{1}{2}$. **6.** 43\frac{1}{2}$. **7.** $98. **8.** 7$\frac{1}{9}$ %.

Page 320. — **3.** $5. **4.** $25. **5.** $220. **6.** $1590. **7.** $980. **8.** $1460. **9.** $3350. **10.** $6350. **11.** $9350. **12.** $25,470. **13.** $16. **14.** $5. **15.** $120. **16.** $300. **17.** $1590. **18.** $950. **19.** $6900. **20.** $10,470. **21.** $9900. **22.** $26,240.

Page 322. — **1.** $1800. **2.** $4475. **3.** $180.80. **4.** $56.25. **5.** $1400 **6.** $583.20. **7.** $75. **8.** $1050. **9.** $5252.31.

Page 324. — **1.** *a.* 2,826,492; *b.* 3,683,841; *c.* $32,858.76; *d.* $18,292.52. **2.** *a.* 71$\frac{3}{8}$; *b.* 67$\frac{11}{30}$; *c.* 147$\frac{15}{16}$; *d.* 68$\frac{1}{2}$. **3.** *a.* 176.7923; *b.* 1830.007; *c.* 1732.6866; *d.* 1707.0909. **4.** *a.* 9 ft. 3 in.; *b.* 12 pk. 2 qt.; *c.* 8 lb. 7 oz.; *d.* 10 doz. **5.** *a.* 278,623; *b.* 132,552; *c.* $3885.47; *d.* $811.18. **6.** *a.* 16$\frac{1}{2}$; *b.* 20$\frac{5}{8}$; *c.* 12$\frac{7}{16}$; *d.* 16. **7.** *a.* 952.5291; *b.* 37.7402; *c.* 517.9238; *d.* 198.256.

Page 325. — **8.** *a.* 4 yd.; *b.* 312 rd.; *c.* 6 wk. 4 da. **9.** *a.* 8,331,126; *b.* $\frac{15}{16}$; *c.* 15.62022; *d.* $13,798.70. **10.** *a.* 1,726,102; *b.* $\frac{21}{16}$; *c.* .034935; *d.* 336. **11.** *a.* $542,124; *b.* 336$\frac{2}{3}$; *c.* .000399; *d.* 25.30572. **12.** *a.* 131; *b.* $\frac{5}{8}$; *c.* 35; *d.* 1 ft. 1$\frac{1}{2}$ in. **13.** *a.* 1600; *b.* $\frac{1}{4}$; *c.* 2.34; *d.* 2$\frac{1}{2}$ qt. **14.** *a.* $7.021; *b.* 5; *c.* 166; *d.* 5.67. **15.** $\frac{1}{4}$; 4; 2. **16.** $\frac{1}{60}$; 60. **17.** *a.* $\frac{2}{4}$; *b.* $\frac{32}{8}$; *c.* $\frac{15}{4}$; *d.* $\frac{32}{5}$; *e.* $\frac{27}{4}$. **18.** *a.* 8$\frac{1}{3}$; *b.* 2; *c.* 5$\frac{1}{3}$; *d.* 6; *e.* 3$\frac{3}{4}$. **19.** $\frac{1}{3}$; $\frac{2}{3}$; $\frac{3}{3}$; $\frac{4}{3}$; $\frac{5}{3}$. **20.** $\frac{19}{20}$, $\frac{5}{20}$, $\frac{4}{20}$; $\frac{12}{16}$, $\frac{6}{16}$, $\frac{8}{16}$; $\frac{12}{30}$, $\frac{9}{30}$, $\frac{2}{30}$; $\frac{24}{40}$, $\frac{5}{40}$, $\frac{16}{40}$. **21.** *a.* .75, 75 %; *b.* .1875, 18$\frac{3}{4}$ %; *c.* .83$\frac{1}{3}$, 83$\frac{1}{3}$ %; *d.* .58$\frac{1}{3}$, 58$\frac{1}{3}$ %; *e.* .16, 16 %. **22.** *a.* $\frac{3}{10}$; *b.* $\frac{3}{4}$; *c.* $\frac{1}{6}$; *d.* 2$\frac{1}{4}$; *e.* 3$\frac{3}{4}$. **23.** *a.* $\frac{3}{4}$; *b.* $\frac{7}{8}$; *c.* $\frac{5}{6}$; *d.* 1$\frac{1}{2}$; *e.* 3$\frac{3}{8}$.

Page 326. — **24.** *a.* 16 pt.; *b.* 3 lb.; *c.* 4 sq. yd. **25.** *a.* 20 qt.; *b.* 4 T.; *c.* 640 sq. rd. **26.** *a.* 2 gal.; *b.* 10,000 lb.; *c.* 2 cu. yd. **27.** *a.* 2 pk.; *b.* 2 ft.; *c.* 108 cu. ft. **28.** *a.* 4 qt.; *b.* 36 in.; *c.* 3 min. **29.** $41; $15; $15. **30.** $4; $9; $12.50. **31.** $6; $8; $13. **32.** 20 yd.; 160 yd.; 150 yd. **33.** 10 yd.; 56 yd.; 39 yd. **34.** *a.* 5; *b.* $80; *c.* 50 T.; *d.* 1468; *e.* 32,000; *f.* 135,000. **35.** *a.* 7.5; *b.* $50; *c.* 125 cu. in.; *d.* 1378; *e.* 48,000; *f.* 180,000. **36.** *a.* 10; *b.* $100; *c.* 175 gal.; *d.* 1650; *e.* 116,000; *f.* 150,000. **37.** *a.* 25%; *b.* 26$\frac{2}{3}$%; *c.* 60%; *d.* 16$\frac{2}{3}$%. **38.** *a.* 400%; *b.* 375%; *c.* 166$\frac{2}{3}$%; *d.* 3$\frac{1}{3}$%. **39.** *a.* 900; *b.* 200; *c.* $560; *d.* $6400. **40.** *a.* 800; *b.* 400; *c.* $160; *d.* $6000.

Page 327. — **41.** *a.* \$25; *b.* \$10; *c.* \$40. **42.** *a.* \$20; *b.* \$36; *c.* \$28. **43.** *a.* 25% gain; *b.* 50% gain; *c.* $16\frac{2}{3}$% loss. **44.** *a.* 25% loss; *b.* $16\frac{2}{3}$% gain; *c.* 20% loss. **45.** *a.* \$25, \$75; *b.* \$50, \$250; *c.* \$120, \$180. **46.** *a.* \$5, \$35; *b.* \$150, \$450; *c.* \$575, \$1425. **47.** *a.* \$1000; *b.* \$150; *c.* \$13.50. **48.** *a.* \$900; *b.* \$360; *c.* \$340. **49.** *a.* \$2; *b.* \$20; *c.* \$120. **50.** *a.* \$24.30; *b.* \$10.50; *c.* \$450. **51.** *a.* \$60; *b.* \$18; *c.* \$18.45. **52.** *a.* \$3; *b.* \$63.75; *c.* \$12.60. **53.** *a.* \$200; *b.* \$5000; *c.* \$150. **54.** *a.* \$700; *b.* \$6000; *c.* \$15.

Page 329. — **15.** \$33.75. **16.** 331 pupils. **17.** \$1.50. **18.** \$41.11. **19.** $21\frac{3}{5}$ yd. **20.** \$360. **21.** 15 yd. **22.** \$.30.

Page 330. — **23.** \$.25. **24.** \$1500. **25.** \$30.25. **26.** 96 mi. **27.** \$760. **28.** \$32.80. **29.** \$500. **30.** $131\frac{1}{4}$%; $76\frac{4}{11}$%; $131\frac{1}{4}$%; $76\frac{4}{11}$%. **31.** $91\frac{2}{3}$%. **32.** $4\frac{11}{18}$%; $3\frac{3}{4}$%; $.4\frac{6}{11}$%. **33.** \$45; \$135. **34.** 80%; \$600.

Page 331. — **35.** \$5400; \$5600. **36.** \$237.60; $.00\frac{1}{3}$; $.33\frac{1}{3}$. **37.** \$280. **38.** \$126.50. **39.** 75%. **40.** 80%; \$9600. **41.** \$8.40. **42.** $4\frac{1}{5}$%. **43.** 20,000 lb. **44.** $\frac{3}{4}$; 75%. **45.** \$7000. **46.** \$1890.

Page 333. — **47.** 2 lb. protein, 1 lb. fat; 2 lb. protein; 1 lb. fat. **48.** $1\frac{1}{2}$ lb. protein, $1\frac{1}{2}$ lb. fat; 2 lb. protein; $1\frac{1}{2}$ lb. fat. **49.** $1\frac{1}{2}$ lb. protein, 1 lb. fat; 2 lb. protein; 2 lb. fat. **50.** $1\frac{1}{2}$ lb. protein, $1\frac{1}{2}$ lb. fat; 2 lb. protein; $1\frac{1}{2}$ lb. fat. **51.** 1 lb. protein, $2\frac{1}{2}$ lb. fat; $1\frac{1}{2}$ lb. protein; $3\frac{1}{2}$ lb. fat. **52.** 1 lb. protein, no fats; $1\frac{1}{2}$ lb. protein, no fats. **53.** $\frac{1}{2}$ lb. **54.** 1 lb. **55.** 1 lb. **56.** 2 lb. **57.** 7.3 lb. **58.** 4.7 lb. **59.** 9.8 lb. **60.** 5.4 lb. **61.** 6.5 lb. **62.** 1 lb. **63.** 7.8 lb. **64.** $\frac{1}{2}$ lb. **65.** $\frac{1}{2}$ lb. **66.** $23\frac{8}{17}$%. **67.** $33\frac{1}{3}$%. **68.** $66\frac{2}{3}$%.

Page 334. — **70.** 3.86%. **71.** 38.75%. **72.** 39.09%. **73.** 23.87%. **74.** 61.36%. **75.** \$6.27. **76.** $548\frac{4}{7}$%. **77.** 70%. **78.** 4000%. **79.** 4.609%.

Page 335. — **80.** $80\frac{10}{13}$%; $72\frac{17}{35}$%. **81.** First, \$.0057; second, \$.0044. **82.** John, $33\frac{1}{3}$%; Charles, $38\frac{2}{21}$%; James, $28\frac{4}{7}$%. **83.** 300 bu.; 900 bu. **84.** 96%. **85.** $66\frac{2}{3}$%. **86.** 249.573 lb.; below, milk; above, butter. **87.** 243.96248 lb.; below, milk; above, butter. **88.** 237.5856 lb.; above, milk; above, butter.

Page 336. — **89.** 255.19 lb.; above, milk; above, butter. **90.** 228.40424 lb.; below, milk; above, butter. **91.** 245.90032 lb.; above, milk; above, butter. **92.** 196.81872 lb.; below, milk; below, butter. **93.** 227.66247 lb.; above, milk; above, butter. **94.** 189.46162 lb.; below, milk; below, butter. **95.** 169.2758 lb.; below, milk; below, butter. **96.** 362.0241 lb. **97.** 357.5628 lb. **98.** 232.02561 lb. **99.** 419.64966 lb. **100.** 318.23 lb. **101.** 406.30575 lb. **102.** 214.64856 lb. **103.** 237.02448 lb. Average for milk, 6304.7$\frac{1}{2}$ lb.; average for butter fat, 318.43387 lb. **104.** \$506.25; 135 bu. **105.** $28\frac{4}{7}$%. **106.** \$50. **107.** \$2674.

Page 338. — **3.** \$60; \$560. **4.** \$36.75; \$211.75. **5.** \$15; \$140. **6.** \$240; \$1040. **7.** \$25.20; \$305.20. **8.** \$194.13; \$754.13. **9.** \$12; \$612. **10.** \$28.60; \$179.10. **11.** \$15.60; \$255.60. **12.** \$20.40; \$260.40.

Page 341. — **3.** \$20.83. **4.** \$24.06. **5.** \$14.63. **6.** \$80. **7.** \$52.42. **8.** \$22.16.

	6%	6%	7%	7%	8%	8%	10%	10%
9.	\$41.04,	\$221.04	\$47.88,	\$227.88	\$54.72,	\$234.72	\$68.40,	\$248.40.
10.	\$25.82,	\$190.82	\$30.12,	\$195.12	\$34.43,	\$199.43	\$43.04,	\$208.04.

	6%	6%	7%	7%	8%	8%	10%	10%
11.	$ 83.03,	$307.03	$ 96.87,	$320.87	$110.71,	$334.71	$138.38,	$ 362.38.
12.	$ 29.57,	$197.57	$ 34.50,	$202.50	$ 39.42,	$207.42	$ 49 28,	$ 217.28.
13.	$ 36.75,	$286.75	$ 42.88,	$292.88	$ 49,	$299	$ 61.25,	$ 311.25.
14.	$202.50,	$952.50	$236.25,	$986.25	$270,	$1020	$337.50,	$1087.50.
15.	$109.20,	$4309.20	$127.40,	$4327.40	$145.60,	$4345.60	$182,	$4382.
16.	$6937½,	$81,937½	$8093¾,	$83,093¾	$9250,	$84250	$11,562½,	$86,562½.

	6%	5%	4%	4½%	8%	8%
17.	$ 10.58	$ 8.81	$ 7.05	$ 7.93	$ 14.10	$ 5.29
18.	$ 2.84	$ 2.36	$ 1.89	$ 2.13	$ 3.78	$ 1.42
19.	$ 27.46	$ 22.88	$ 18.31	$ 20.59	$ 36.61	$ 13.73
20.	$ 18.43	$ 15.36	$ 12.29	$ 13.82	$ 24.57	$ 9.21
21.	$ 63.98	$ 53.32	$ 42.65	$ 47.99	$ 85.31	$ 31.99
22.	$ 44.63	$ 37.19	$ 29.75	$ 33.47	$ 59.50	$ 22.31
23.	$ 31.90	$ 26.58	$ 21.27	$ 23.93	$ 42.53	$ 15.95
24.	$ 51.94	$ 43.28	$ 34.63	$ 38.96	$ 69.25	$ 25.97
25.	$ 37.98	$ 31.65	$ 25.32	$ 28.49	$ 50.64	$ 18.99
26.	$ 72.23	$ 60.19	$ 48.15	$ 54.17	$ 96.30	$ 36.11
27.	$ 44.28	$ 36.90	$ 29.52	$ 33.21	$ 59.04	$ 22.14
28.	$234.47	$195.39	$156.31	$175.85	$312.62	$117.23
29.	$ 38.40	$ 32	$ 25.60	$ 28.80	$ 51.20	$ 19.20
30.	$ 66.10	$ 55.08	$ 44.06	$ 49.57	$ 88.13	$ 33.05
31.	$ 43.69	$ 36.41	$ 29.13	$ 32.77	$ 58.25	$ 21.85
32.	$ 79.80	$ 66.50	$ 53.20	$ 59.85	$106.40	$ 39.90

Page 342.—

	6%	5%	4%	4½%	8%	3%
33.	$ 120.42	$ 100.35	$ 80.28	$ 90.32	$ 160.56	$ 60.21
34.	$ 190.08	$ 158.40	$126.72	$142.56	$ 253.44	$ 95.04
35.	$ 149.86	$ 124.88	$ 99.90	$112.39	$ 199.81	$ 74.93
36.	$ 106.08	$ 88.40	$ 70.72	$ 79.56	$ 141.44	$ 53.04
37.	$ 24.68	$ 20.56	$ 16.45	$ 18.51	$ 32.90	$ 12.34
38.	$1291.30	$1076.08	$860.87	$968.48	$1721.73	$645.65
39.	$ 34.86	$ 29.05	$ 23.24	$ 26.15	$ 46.48	$ 17.43
40.	$ 15.39	$ 12.83	$ 10.26	$ 11.54	$ 20.52	$ 7.70
41.	$ 69.75	$ 58.12	$ 46.50	$ 52.31	$ 92.99	$ 34.87
42.	$ 1.17	$.98	$.78	$.88	$ 1.56	$.59
43.	$ 190.23	$ 158.52	$126.82	$142.67	$ 253.64	$ 95.11
44.	$ 189.47	$ 157.89	$126.31	$142.10	$ 252.62	$ 94.73
45.	$ 82.42	$ 68.68	$ 54.95	$ 61.82	$ 109.89	$ 41.21
46.	$ 113.12	$ 94.27	$ 75.41	$ 84.84	$ 150.83	$ 56.56
47.	$ 40.72	$ 33.94	$ 27.15	$ 30.54	$ 54.30	$ 20.36
48.	$ 162.83	$ 135.69	$108.55	$122.12	$ 217.11	$ 81.42
49.	$ 5.30	$ 4.42	$ 3.53	$ 3.98	$ 7.07	$ 2.65
50.	$ 11.22	$ 9.35	$ 7.48	$ 8.42	$ 14.96	$ 5.61
51.	$ 4.29	$ 3.57	$ 2.86	$ 3.22	$ 5.71	$ 2.14
52.	$ 241.25	$ 201.04	$160.83	$180.94	$ 321.67	$120.63
53.	$ 34.49	$ 28.74	$ 22.99	$ 25.87	$ 45.99	$ 17.24
54.	$ 3.71	$ 3.09	$ 2.47	$ 2.78	$ 4.94	$ 1.85
55.	$ 15.07	$ 12.56	$ 10.04	$ 11.30	$ 20.09	$ 7.53

	6%	5%	4%	4½%	8%	3%
56.	$115.26	$96.05	$76.84	$86.44	$153.68	$57.63
57.	$ 38.85	$32.38	$25.90	$29.14	$ 51.80	$19.43
58.	$ 66.14	$55.12	$44.10	$49.61	$ 88.19	$33.07
59.	$ 32.01	$26.67	$21.34	$24.01	$ 42.68	$16
60.	$ 24.13	$20.11	$16.08	$18.10	$ 32.17	$12.06

Page 344. — **26.** $53.25. **27.** $29.58. **28.** $62.78. **29.** $48.94. **30.** $26.32. **31.** $33.48. **32.** $245.33. **33.** $40.73. **34.** $77.70. **35.** $131.87. **36.** $147.50. **37.** $91.52. **38.** $331.25. **39.** $212.80. **40.** $153.13.

Page 345. — **41.** $90.89. **42.** $77.06. **43.** $195.96.

2–127.

	3%	4%	4½%	5%	2½%	5½%	3¾%
(26)	$ 26.63	$ 35.50	$ 39.94	$ 44.38	$ 22.19	$ 48.81	$ 33.28
(27)	$ 14.79	$ 19.72	$ 22.18	$ 24.65	$ 12.32	$ 27.11	$ 18.48
(28)	$ 31.39	$ 41.85	$ 47.08	$ 52.31	$ 26.16	$ 57.55	$ 39.24
(29)	$ 24.47	$ 32.63	$ 36.70	$ 40.78	$ 20.39	$ 44.86	$ 30.58
(30)	$ 13.16	$ 17.55	$ 19.74	$ 21.94	$ 10.97	$ 24.13	$ 16.45
(31)	$ 16.74	$ 22.32	$ 25.10	$ 27.90	$ 13.95	$ 30.69	$ 20.92
(32)	$122.67	$163.55	$184	$204.44	$102.22	$224.89	$153.33
(33)	$ 20.37	$ 27.15	$ 30.55	$ 33.94	$ 16.97	$ 37.35	$ 25.46
(34)	$ 38.85	$ 51.80	$ 58.28	$ 64.75	$ 32.38	$ 71.23	$ 48.56
(35)	$ 65.93	$ 87.91	$ 98.90	$109.89	$ 54.94	$120.88	$ 82.42
(36)	$ 73.75	$ 98.33	$110.63	$122.92	$ 61.46	$135.21	$ 92.19
(37)	$ 45.76	$ 61.01	$ 68.64	$ 76.27	$ 38.13	$ 83.89	$ 57.20
(38)	$165.63	$220.83	$248.44	$276.04	$138.02	$303.65	$207.03
(39)	$106.40	$141.87	$159.60	$177.33	$ 88.67	$195.07	$133
(40)	$ 76.56	$102.08	$114.84	$127.61	$ 63.80	$140.36	$ 95.70
(41)	$ 45.45	$ 60.59	$ 68.17	$ 75.74	$ 37.87	$ 83.32	$ 56.81
(42)	$ 38.53	$ 51.37	$ 57.79	$ 64.21	$ 32.11	$ 70.63	$ 48.16
(43)	$ 97.98	$130.64	$146.97	$163.30	$ 81.65	$179.63	$122.47

128. $20.70. **129.** $44.38. **130.** $11.58.

Page 346. — **1.** $73. **2.** $72; $1 less.

Page 347. — **3.** 216 da. **4.** 277 da. **5.** $35.11.

Page 348. — **1.** $.44. **2.** $1.36. **3.** $1.32. **4.** $2.06. **5.** $1.26; $1.51. **6.** $9.86; $7.40. **7.** $8.63; $12.95. **8.** $5.47. **9.** $5.74. **10.** $60.59.

Page 349. — **2.** 6%. **3.** 5%. **4.** 8%. **5.** 5%. **6.** 10%; 5%; 4%.

Page 350. — **2.** 4 yr. **3.** 2 yr. **4.** 3 yr. **6.** 16 yr. 8 mo.; 10 yr.; 25 yr.

2. $2000. **3.** $3000. **4.** $3000. **5.** $560. **6.** $770.

Page 355. — **11.** $2687.50.

Page 356. — **12.** April 30, 1914; $559.17. **13.** $1015. **14.** $888.13; $887.95. **15.** June 1, 1914; $1512.50.

2. $292.50. **3.** $1375. **4.** $306.25.

Page 358. — **1.** $2127.48. **2.** $739.10.

Page 359. — **4.** $5444.26. **5.** $2141.37. **6.** $1363.66. **7.** $701.43.

Page 362. — **3.** $2; $198. **4.** $2.19; $122.81. **5.** $1.17; $173.83. **6.** $5.63; $244.37. **7.** $1.13; $148.87. **8.** $4.17; $995.83. **9.** $11.25; $1488.75. **10.** $24; $1576. **11.** $17.19; $2482.81. **12.** $30; $2970. **13.** April 5, 1914. **14.** Feb. 3, 1911; 55 da.; $4.88; $451.87. **15.** April 5, 1914; $7.73; $610.27.

Page 363. — **16.** $2.63; $754.87. **17.** $.62; $177.44. **18.** $1.28; $126.22. **19.** $1.53; $227.97. **20.** $2.43; $500.90. **21.** $3.49; $1000.68. **22.** $24.83; $2500.17. **23.** $24.85; $1793.15. **24.** $17.61; $3504.27. **25.** $50.33; $5989.67. **26.** $39,723. **27.** $8,075,540.20. **28.** $14,674,649.20.

Page 365. — **2.** $3370.80; $370.80. **3.** $4329.73.

Page 366. — **5.** $1819.98. **6.** $657.43. **7.** $1900.03. **8.** $512.11.

Page 368. — **1.** Feb. 1, 1914; Feb. 1, 1915; $.40. **2.** Feb. 1, 1914; March 1, 1914; April 1, 1914. **3.** $1.70. **4.** $.30; $1. **5.** $6.25. **6.** $4.38.

Page 373. — **1.** $2.38; $2.60; $8.08; $55.95; $98.90. **2.** $7.35. **4.** $4761.88. **5.** $354.71. **7.** $20,737.50. **8.** $1995. **9.** $2996.25. **10.** $11,880.

Page 374. — **11.** $479.52. **12.** $2502.50. **13.** $5005. **14.** $3506 30. **15.** $50 check: £10 2*s.* 11*d.*, 256.25 francs, 206.25 marks; $10 check: £2 7*d.*, 51.25 francs, 41.25 marks; $100 check: £20 5*s.* 10*d.*, 512.5 francs, 412.5 marks.

Page 377. — **2.** $250. **3.** $5500.

Page 378. — **1–24.** (1) $710; (2) $460.63; (3) $928.75; (4) $1115.63; (5) $241.25; (6) $13,012.50; (7) $542.50; (8) $3183.75; (9) $7418.75; (10) $10,712.50; (11) $860; (12) $280.63; (13) $4100; (14) $3700; (15) $8837.50; (16) $2958.75; (17) $1930; (18) $5475; (19) $10,412.50; (20) $6420; (21) $5143.75; (22) $3440; (23) $10,162.50; (24) $2718.75. **25–48.** (1) $707.50; (2) $459.38; (3) $926.25; (4) $1114.38; (5) $238.75; (6) $12,987.50; (7) $537.50; (8) $3176.25; (9) $7406.25; (10) $10,687.50; (11) $857.50; (12) $279.38; (13) $4090; (14) $3687.50; (15) $8812.50; (16) $2951.25; (17) $1925; (18) $5460; (19) $10,387.50; (20) $6400; (21) $5131.25; (22) $3430; (23) $10,137.50; (24) $2711.25.

Page 379. — **49–60.** (13) $120; (14) $200; (15) $450; (16) $120; (17) $80; (18) $240; (19) $500; (20) $280; (21) $250; (22) $160; (23) $500; (24) $120. **62.** (13) 2.9%; (14) 5.4%; (15) 5.1%; (16) 4.1%; (17) 4.1%; (18) 4.4%; (19) 4.8%; (20) 4.4%; (21) 4.9%; (22) 4.7%; (23) 4.9%; (24) 4.4%. **63.** $1605. **64.** $40. **65.** $150; 2.9%.

Page 382. — **30.** $1100. **31.** 12%. **32.** $1.50. **33.** $144. **34.** $8000. **35.** $15,887.50. **36.** 218 da. **37.** $12.09. **38.** $115.84. **39.** $200 gain; $3\frac{41}{53}$% gain. **40.** 15% gain. **41.** $1045.50; $4100. **42.** $956.25.

Page 383. — **43.** $75. **44.** $1.51. **45.** $525. **46.** 9500%; 95 pk. **47.** $1\frac{9}{16}$% loss. **48.** 16¢ a dozen; 15¢ a dozen. **49.** $1.80; $1.08; 28%. **50.** $2640, or $66 an acre. **51.** $100,000. **52.** $4500. **53.** $42\frac{6}{7}$%.

Page 384. — **54.** $20,000. **55.** $25 loss. **56.** 100%. **57.** 17%. **58.** $12\frac{1}{2}$%. **59.** $2000. **60.** $27.88. **61.** $75. **62.** 20%. **63.** $\frac{3}{4}$%; $30; $3970.

Page 385. — **64.** $1200. **65.** .02%; $3.94. **66.** $320. **67.** $412.53. **69.** $300.73. **70.** $1521.52. **71.** Paying $500 in 6 mo. will cost him $5 less. **72.** $50; $45; $40. **73.** 5%. **74.** $341.25.

Page 392. — **9.** $x+4=6$. **10.** $x-3=6$. **11.** $5x=25$. **12.** $\frac{x}{4}=4$.

Page 393. — **13.** $x+7=25$. **14.** $x+10=30$. **15.** $17-x=5$. **16.** $2x+4=50$. **17.** $\frac{x}{4}-3=10$. **18.** $\frac{x}{2}+\frac{x}{4}=8$. **19.** $x+x+4=35$. **21.** A, $\frac{1}{4}$; B, $\frac{1}{6}$; $\frac{5}{12}$; $\frac{1}{x}$; $\frac{1}{4}+\frac{1}{6}=\frac{1}{x}$. **22.** $\frac{d}{c}$; $\frac{c}{d}$; $\frac{2c}{d}$. **23.** $2x$; $4x$; $x+2x$; $x+2x+4x$; $2x-x$.

Page 394. — **24.** $25-8$; $25-b$. **25.** $a-6$. **26.** Father, $3x$; son, x; father, $3x+10$; son, $x+10$; $2x+20$; $3x+10=2x+20$.

Page 395. — **3.** 2. **4.** 20. **5.** 2. **6.** 4. **7.** 8. **8.** 30. **11.** 5. **12.** 6. **13.** 9. **14.** 6. **15.** 7. **16.** 3.

Page 396. — **19.** 20. **20.** 12. **21.** 25. **22.** 36. **23.** 12. **24.** 35.

Page 397. — **2.** 783. **3.** 13 yr. **5.** 100; 500. **6.** $194, horse; $80, carriage. **7.** 120. **8.** 60. **9.** 50. **10.** 70. **11.** 18. **12.** 222 boys, 278 girls.

Page 398. — **16.** 60 sq. in. **18.** A, $5000; B, $15,000; C, $10,000. **19.** Shoes, $4; gloves, $2; dress, $21. **20.** First, 8; second, 3; third, 13. **21.** First, 6; second, 3; third, 24. **22.** 27; $42-x$.

Page 399. — **23.** $18-x$; 6, 12. **24.** $\frac{1}{b}$, $\frac{a}{b}$; $\frac{1}{b}$, $\frac{c}{b}$. **25.** 80. **26.** 19, 23, 15. **27.** 40. **28.** 30, 90, 120. **29.** 762. **30.** 340. **31.** 25. **32.** 131. **33.** 50. **34.** 25. **35.** 25, 75. **36.** Mother, 32; daughter, 8. **37.** First, 5; second, 10; third, 15.

Page 402. — **2.** 225. **3.** 625. **4.** 1225. **5.** 1600. **6.** 256. **7.** 361. **8.** 729. **9.** 7056. **10.** 22,500. **11.** 62,500.

Page 406. — **8.** 48. **9.** 45. **10.** 16. **11.** 65. **12.** 85. **13.** 64. **14.** 27. **15.** 34. **16.** 72. **17.** 365. **18.** 324. **19.** 723. **20.** 314. **21.** 625. **22.** 702. **23.** 506. **24.** 250. **25.** 700. **26.** 6.72. **27.** 54.8. **28.** 5.25. **29.** 6.31. **30.** 7.13. **31.** 8.14. **32.** 18.57+. **33.** 2.23+. **34.** $\frac{4}{5}$. **35.** $\frac{3}{4}$.

Page 407. — **1.** 480 rd. **2.** $2560. **3.** $492; $246 more. **4.** 240 rd.; 40 rd. more. **6.** 6 ft., 18 ft. **7.** 27,556 hills. **8.** $2\frac{2}{11}$ rd., $4\frac{4}{11}$ rd.

Page 412. — **4.** $142\frac{1}{2}$ sq. in. **5.** $56\frac{7}{8}$ A. **6.** 100 sq. rd. **7.** $16\frac{1}{4}$ ft. **8.** $13\frac{1}{3}$ yd.

Page 413. — **1.** 150 sq. in. **2.** 5304 sq. in. **3.** $10\frac{25}{48}$ sq. ft. **4.** $35\frac{1}{16}$ A. **5.** $22\frac{1}{2}$ bd. ft. **6.** Areas on each side of walk, 180 sq. ft.; 240 sq. ft.; area of entire lot, 480 sq. ft.; area of walk, 60 sq. ft.

Page 415. —**2.** 69 in. **3.** 25 ft. **4.** 240 mi. **5.** 46.87 rd. **6.** 89 ft. **7.** 36 ft. **8.** 60 ft.

Page 417. —**8.** 3087½ sq. rd. **9.** 175 sq. in. **10.** 810 sq. in.

Page 418. —**2.** 290.47 sq. rd. **3.** 54 sq. in.; ⅛. **4.** 85.32 sq. ft. **5.** 187.08 sq. rd. **6.** 83.9 sq. yd. **7.** 38.9 sq. in.

Page 419. —**1.** *EB*, 14.4 ft.; *EC*, 16.9 ft.; *ED*, 19.2 ft.; *EB*, 15.4 ft.; *EC*, 17.9 ft.; *ED*, 20.2 ft.; 12 ft. **2.** 38 ft. **3.** *EBF*, 1170.4 sq. ft.; *ECF*, 1360.4 sq. ft.; *EDF*, 1535.2 sq. ft. **4.** *EBF*, 47 bunches; *ECF*, 55 bunches; *EDF*, 62 bunches.

Page 420. —**6.** ⅛ pitch. **7.** 15 ft. **8.** $\frac{5}{16}$ pitch. **9.** 52 bunches. **10.** 68 bunches. **11.** 57 bunches.

Page 421. —**4.** 78.54 sq. ft.; 1256.64 sq. ft.

Page 422. —**5.** 31,416 sq. ft. **6.** 751 bricks.

Page 423. —**7.** 314.16. **8.** 200. **9.** 43.9824. **10.** 78.54. **11.** 11,309.76. **12.** 16.6. **13.** 4. **14.** 25.1328. **15.** 14.2 rd. **16.** 1963.5. **17.** 3117.2526 sq. ft. **18.** 78.54. **19.** $D = 57.29$ rd.; $C = 179.98$ rd. **20.** $A = 19.635$ sq. ft.; 16 circles. **21.** 33 in. **22.** The square flue is 54.9376 sq. in. larger.

Page 424. —**23.** 36.53 sq. in. **24.** 7.0686 sq. in. **25.** 314.16 sq. ft.

Page 427. —**1.** 112 sq. in.; 100 sq. in. **2.** 240 sq. in.; 252 sq. in. **3.** 144 sq. in.; 176 sq. in. **4.** 603.1872 sq. in.; 829.3824 sq. in. **5.** 1005.312 sq. in.; 1407.4368 sq. in. **6.** 480 sq. in.; 624 sq. in. **7.** 324 sq. in. **8.** 1256.64 sq. in.; 1570.8 sq. in.

Page 428. —**9.** 314.16 sq. in. **10.** Areas are: sphere, 1256.64 sq. in.; cube, 2400 sq. in.; and circle, 314.16 sq. in.

Page 429. —**15.** 512 cu. in.; 4096 cu. in. **16.** 108 cu. ft.

Page 430. —**18.** $333.34. **19.** 4693⅓ cu. yd. **20.** 41$\frac{11}{27}$ cu. yd. **22.** 5266⅔ bbl. **23.** 26$\frac{2}{11}$ bbl. **24.** Approximate, 768 bu.; exact, 771⅓ bu. **25.** 603.1872 cu. in. **26.** 791.6832 cu. in.; 904.7808 cu. in. **27.** 100.21 lb.

Page 431. —**28.** 69.813 lb. **29.** 201.0624 cu. in. **30.** Approximate, 515.41 gal.; exact, 514.08 gal. **31.** Approximate, 2250 gal.; exact, 2244$\frac{4}{7}$ gal. **32.** 18,750 lb. **33.** 5 ft. **34.** 57 loads. **35.** 27.6 ft. **36.** 3640 bricks. **37.** 22.2 ft.

Page 432. —**6.** $3.20.

Page 433. —**8.** $495.93. **9.** $16.19. **10.** $4.80. **11.** $213.50. **12.** $24 crosswise; $27 lengthwise. **13.** $50 lengthwise; $52.19 crosswise.

Page 434. —**14.** $1287.60. **15.** $1650. **16.** 1,474,560 A. **17.** 2½ mi. **18.** $103,092. **19.** $195.56 cheaper to pave. **20.** 2340 sq. ft. **21.** 19⅛ A. **22.** 2062.5 sq. rd. **23.** 20 in. **24.** 2945 sq. rd.

Page 435. —**25.** $2000. **26.** 54.9376 sq. in. **27.** 150.7968 sq. in. **28.** 12,063.744 lb. **29.** 93.4626 sq. in. **30.** 1.12 in. **31.** 216 sq. in.; 864 sq. in. **32.** 761$\frac{1}{16}$ cu. ft. **33.** 102⅔ bu. **34.** 199 rows; 265 rows. **35.** 864 shocks; 124,416 hills. **36.** Owner, $366.61; tenant, $244.40.

Page 436. — **37.** 392.7 cu. in. **38.** Approximate, 100 bu.; exact, $100\frac{25}{56}$ bu. **39.** 381.1808 cu. in. **40.** 420 sq. in. **41.** $32\frac{5}{24}$ sq. yd. **42.** 240 sq. in. **43.** 471.24 sq. in.; 628.32 sq. in. **44.** 27.92 sq. yd. **45.** 432 sq. in. **46.** 508.94 sq. in. **47.** 3.1416 sq. in.; 314.16 sq. in. **48.** $4.72.

Page 445. — **89.** *a.* 4,834,062; *b.* 5,482,783; *c.* $46,448.56; *d.* $51,924.02. **90.** *a.* 59.1351; *b.* 3828.5818; *c.* 15,253.7026; *d.* 1002.7044. **91.** *a.* $224\frac{1}{5}$; *b.* $45\frac{1}{10}$; *c.* $1196\frac{1}{2}$; *d.* $116\frac{1}{12}$; *e.* $117\frac{29}{80}$. **92.** *a.* 29 ft. 2 in.; *b.* 30 pk. 1 qt.; *c.* 40 lb. 3 oz. **93.** *a.* 8 gal. 1 qt.; *b.* 8 yd.; *c.* 61 mi. 130 rd.

Page 446. — **94.** *a.* 76,623; *b.* 364,121; *c.* $4148.74; *d.* $321.71. **95.** *a.* 200.0754; *b.* 291.0857; *c.* 561.9424; *d.* 1078.195. **96.** *a.* $96\frac{7}{8}$; *b.* $95\frac{1}{2}$; *c.* $25\frac{15}{16}$; *d.* $248\frac{5}{48}$. **97.** *a.* 1 ft. 10 in.; *b.* 8 lb. 14 oz.; *c.* 8 wk. 5 da. **98.** 1 yr. 4 mo. 19 da.; 1 yr. 10 mo. 5 da. **99.** 1 yr. 4 mo. 24 da.; 1 yr. 5 mo. 13 da. **100.** *a.* $.22, $1.22; *b.* $3.46, $8.46; *c.* $2.64, $7.64; *d.* $.02, $5.02; *e.* $4.25, $9.25. **101.** *a.* $.35, $1.35; *b.* $2.25, $7.25; *c.* $4.84, $9.84; *d.* $1.48, $6.48; *e.* $3.02, $8.02. **102.** *a.* $.61, $1.61; *b.* $1.35, $6.35; *c.* $4.46, $9.46; *d.* $2.61, $7.61; *e.* $2.84, $7.84. **103.** *a.* $.03, $1.03; *b.* $.06, $5.06; *c.* $3.75, $8.75; *d.* $.17, $5.17; *e.* $.55, $5.55.

Page 447. — **104.** *a.* 10,464,784; *b.* $\frac{1}{8}$; *c.* 6.9103125. **105.** *a.* 2,214,432; *b.* $\frac{7}{10}$; *c.* .005184. **106.** *a.* 89,434,849; *b.* $189\frac{73}{125}$; *c.* 6.35408. **107.** *a.* $68,369.20; *b.* $259\frac{7}{20}$; *c.* 4218.3515. **108.** *a.* $282,638.925; *b.* $2823\frac{3}{64}$; *c.* $.9531. **109.** *a.* 6246.468; *b.* $430\frac{68}{75}$; *c.* $7.7051. **110.** *a.* 6839.02575; *b.* $666\frac{8}{200}$; *c.* 7.5. **111.** *a.* 13,089.049515; *b.* $296\frac{57}{125}$; *c.* 50. **112.** *a.* 104; *b.* $1\frac{1}{4}$; *c.* 7 in. **113.** *a.* $264\frac{3}{457}$; *b.* $1\frac{1}{8}$; *c.* 2 qt. **114.** *a.* $9.57; *b.* 3; *c.* 9 oz. **115.** *a.* 123; *b.* $1\frac{7}{16}$; *c.* $4\frac{1}{2}$ da. **116.** *a.* 3440; *b.* $3\frac{103}{128}$; *c.* 8 units. **117.** *a.* 3102; *b.* $9\frac{17}{35}$; *c.* 4 mo. **118.** *a.* 358,000; *b.* $4\frac{1}{2}$; *c.* 3 qt. **119.** *a.* 212; *b.* $6\frac{6}{35}$; *c.* 3 ft. **120.** *a.* $\frac{1}{4}$; *b.* $\frac{15}{16}$; *c.* $\frac{1}{16}$. **121.** *a.* 4; *b.* $\frac{16}{15}$ or $1\frac{1}{15}$; *c.* $\frac{8}{3}$ or $2\frac{2}{3}$. **122.** *a.* $\frac{1}{3}$; *b.* $\frac{6}{7}$; *c.* $\frac{1}{3}$. **123.** *a.* 3; *b.* $\frac{7}{8}$ or $1\frac{1}{7}$; *c.* 8.

Page 448. — **124.** *a.* $\frac{7}{4}$; *b.* $\frac{59}{4}$; *c.* $\frac{123}{8}$; *d.* $\frac{250}{3}$. **125.** *a.* $\frac{14}{3}$; *b.* $\frac{125}{8}$; *c.* $\frac{100}{3}$; *d.* $\frac{245}{2}$. **126.** *a.* $\frac{23}{4}$; *b.* $\frac{50}{3}$; *c.* $1\frac{23}{24}$; *d.* $\frac{98}{5}$. **127.** *a.* $\frac{59}{8}$; *b.* $\frac{305}{2}$; *c.* $\frac{175}{2}$; *d.* $\frac{219}{5}$. **128.** *a.* $16\frac{3}{8}$; *b.* $12\frac{1}{4}$; *c.* 28; *d.* $8\frac{3}{5}$; *e.* $5\frac{1}{6}$. **129.** *a.* 7; *b.* $2\frac{7}{8}$; *c.* 9; *d.* $4\frac{1}{2}$; *e.* $5\frac{5}{12}$. **130.** *a.* $4\frac{13}{15}$; *b.* $\frac{4}{5}$; *c.* 5; *d.* 3; *e.* $3\frac{1}{11}$. **131.** *a.* $\frac{7}{8}$; *b.* $\frac{2}{3}$; *c.* $\frac{2}{3}$; *d.* $\frac{1}{2}$; *e.* $\frac{4}{5}$; *f.* $\frac{2}{5}$. **132.** *a.* $\frac{3}{5}$; *b.* $\frac{3}{7}$; *c.* $\frac{3}{8}$; *d.* $\frac{1}{4}$; *e.* $\frac{1}{5}$; *f.* $\frac{1}{2}$. **133.** *a.* $\frac{2}{3}$; *b.* $\frac{4}{7}$; *c.* $\frac{3}{5}$; *d.* $\frac{1}{4}$; *e.* $\frac{3}{8}$; *f.* $\frac{5}{12}$. **134.** *a.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{3}{12}$; *b.* $\frac{20}{30}$, $\frac{18}{30}$, $\frac{9}{30}$; *c.* $\frac{5}{30}$, $\frac{12}{30}$, $\frac{9}{30}$. **135.** *a.* $\frac{45}{60}$, $\frac{10}{60}$, $\frac{24}{60}$; *b.* $\frac{18}{48}$, $\frac{20}{48}$, $\frac{9}{48}$; *c.* $\frac{9}{12}$, $\frac{5}{12}$, $\frac{10}{12}$. **136.** *a.* $\frac{4}{8}$, $\frac{6}{8}$, $\frac{3}{8}$; *b.* $\frac{8}{16}$, $\frac{6}{16}$, $\frac{8}{16}$; *c.* $\frac{14}{24}$, $\frac{15}{24}$; $\frac{8}{24}$. **137.** *a.* .5, 50%; *b.* .66$\frac{2}{3}$, 66$\frac{2}{3}$%; *c.* .4, 40%; *d.* .83$\frac{1}{3}$, 83$\frac{1}{3}$%; *e.* .625, 62$\frac{1}{2}$%; *f.* .58$\frac{1}{3}$, 58$\frac{1}{3}$%; *g.* .12, 12%. **138.** *a.* .25, 25%; *b.* .16$\frac{2}{3}$, 16$\frac{2}{3}$%; *c.* .6, 60%; *d.* .14$\frac{2}{7}$, 14$\frac{2}{7}$%; *e.* .875, 87$\frac{1}{2}$%; *f.* .1875, 18$\frac{3}{4}$%; *g.* .14, 14%. **139.** *a.* .75, 75%; *b.* .0625, 6$\frac{1}{4}$%; *c.* .2, 20%; *d.* .125, 12$\frac{1}{2}$%; *e.* .41$\frac{2}{3}$, 41$\frac{2}{3}$%; *f.* .91$\frac{2}{3}$, 91$\frac{2}{3}$%; *g.* .85, 85%. **140.** *a.* .33$\frac{1}{3}$, 33$\frac{1}{3}$%; *b.* .08$\frac{1}{3}$, 8$\frac{1}{3}$%; *c.* .8, 80%; *d.* .375, 37$\frac{1}{2}$%; *e.* .3125, 31$\frac{1}{4}$%; *f.* .4375, 43$\frac{3}{4}$%; *g.* .44, 44%.

Page 449. — **141.** *a.* 20%, $\frac{1}{5}$; *b.* 12 5%, $\frac{1}{8}$; *c.* 16$\frac{2}{3}$%, $\frac{1}{6}$; *d.* 175%, $1\frac{3}{4}$; *e.* 112.5%, $1\frac{1}{8}$. **142.** *a.* 40%, $\frac{2}{5}$; *b.* 37.5%, $\frac{3}{8}$; *c.* 33$\frac{1}{3}$%, $\frac{1}{3}$; *d.* 125%, $1\frac{1}{4}$; *e.* 133$\frac{1}{3}$%, $1\frac{1}{3}$. **143.** *a.* 50%, $\frac{1}{2}$; *b.* 62.5%, $\frac{5}{8}$; *c.* 66$\frac{2}{3}$%, $\frac{2}{3}$; *d.* 150%, $1\frac{1}{2}$; *e.* 137.5%, $1\frac{3}{8}$. **144.** *a.* 60%, $\frac{3}{5}$; *b.* 87.5%, $\frac{7}{8}$; *c.* 83$\frac{1}{3}$%, $\frac{5}{6}$; *d.* 140%, $1\frac{2}{5}$; *e.* 187.5%, $1\frac{7}{8}$. **145** *a.* 25%, $\frac{1}{4}$; *b.* 6$\frac{1}{4}$%, $\frac{1}{16}$; *c.* 8$\frac{1}{3}$%, $\frac{1}{12}$; *d.* 160%, $1\frac{3}{5}$; *e.* 166$\frac{2}{3}$%, $1\frac{2}{3}$. **146.** *a.* 8 qt.; *b.* 80 oz.; *c.* 45 sq. ft. **147.** *a.* 10 pt.; *b.* 5 lb.;

c. 3 sq. yd. **148.** *a.* 16 qt.; *b.* 2 T.; *c.* 480 sq. rd. **149.** *a.* 6 gal.; *b.* 6000 lb.; *c.* 2 cu. yd. **150.** *a.* 5 pk.; *b.* 3 ft.; *c.* 135 cu. ft. **151.** *a.* 40 qt.; *b.* 48 in.; *c.* 2 min. **152.** *a.* 25 bu.; *b.* 2 yd.; *c.* 300 min. **153.** *a.* 64 pk.; *b.* 6 yd.; *c.* 3 hr. **154.** *a.* $5; *b.* $40; *c.* $75. **155.** *a.* $13; *b.* $18; *c.* $31.25. **156.** *a.* $21; *b.* $6; *c.* $39.20. **157.** *a.* $10; *b.* $63; *c.* $87.50. **158.** *a.* $24; *b.* $25; *c.* $91. **159.** *a.* $3; *b.* $4; *c.* $120. **160.** *a.* $11; *b.* $15; *c.* $132. **161.** *a.* $28; *b.* $7; *c.* $88.

Page 450.—162. *a.* 20 yd.; *b.* 108 yd. **163.** *a.* 160 yd.; *b.* 108 yd. **164.** *a.* 48 yd.; *b.* 102 yd. **165.** *a.* 220 yd.; *b.* 1536 yd. **166.** *a.* 136 yd.; *b.* 70 yd. **167.** *a.* 40 yd.; *b.* $\frac{2}{15}$ yd. **168.** *a.* 40 yd.; *b.* $\frac{1}{5}$ yd. **169.** *a.* 56 yd.; *b.* $\frac{1}{4}$ yd. **170.** *a.* 10; *b.* 11; *c.* $880; *d.* 91; *e.* $100. **171.** *a.* 15; *b.* 20; *c.* $960; *d.* 112; *e.* $130. **172.** *a.* 20; *b.* 31; *c.* $720; *d.* 315; *e.* $139. **173.** *a.* 10; *b.* 20; *c.* $150; *d.* 150; *e.* $220. **174** *a.* 52; *b.* 30; *c.* $180; *d.* 200; *e.* $300. **175.** *a.* 70; *b.* 40; *c.* $270; *d.* 705; *e.* $528. **176.** 6%: (170) *a.* 3; *b.* 5.28; *c.* $39.60; *d.* 6.24; *e.* $36. (171) *a.* 4.5; *b.* 9.6; *c.* $43.20; *d.* 7.68; *e.* $46.80. (172) *a.* 6; *b.* 14.88; *c.* $32.40; *d.* 21.6; *e.* $50.04. (173) *a.* 7.2; *b.* 19.2; *c.* $48; *d.* 10.8; *e.* $19.80. (174) *a.* 37.44; *b.* 28.8; *c.* $57.60; *d.* 14.4; *e.* $27. (175) *a.* 50.4; *b.* 38.4; *c.* $86.40; *d.* 50.76; *e.* $47.52. 5%: (170) *a.* 2.5; *b.* 4.4; *c.* $33; *d.* 5.2; *e.* $30. (171) *a.* 3.75; *b.* 8; *c.* $36; *d.* 6.4; *e.* $39. (172) *a.* 5; *b.* 12.4; *c.* $27; *d.* 18; *e.* $41.70. (173) *a.* 6; *b.* 16; *c.* $40; *d.* 9; *e.* $16.50. (174) *a.* 31.2; *b.* 24; *c.* $48; *d.* 12; *e.* $22.50. (175) *a.* 42; *b.* 32; *c.* $72; *d.* 42.3; *e.* $39.60. $4\frac{1}{2}$%: (170) *a.* 2.25; *b.* 3.96; *c.* $29.70; *d.* 4.68; *e.* $27. (171) *a.* 3.375; *b.* 7.2; *c.* $32.40; *d.* 5.76; *e.* $35.10. (172) *a.* 4.5; *b.* 11.16; *c.* $24.30; *d.* 16.2; *e.* $37.53. (173) *a.* 5.4; *b.* 14.4; *c.* $36; *d.* 8.1; *e.* $14.85. (174) *a.* 28.08; *b.* 21.6; *c.* $43.20; *d.* 10.8; *e.* $20.25. (175) *a.* 37.8; *b.* 28.8; *c.* $64.80; *d.* 38.07; *e.* $35.64. $3\frac{1}{2}$%: (170) *a.* 1.75; *b.* 3.08; *c.* $23.10; *d.* 3.64; *e.* $21. (171) *a.* 2.625; *b.* 5.6; *c.* $25.20; *d.* 4.48; *e.* $27.30. (172) *a.* 3.5; *b.* 8.68; *c.* $18.90; *d.* 12.6; *e.* $29.19. (173) *a.* 4.2; *b.* 11.2. *c.* $28; *d.* 6.3; *e.* $11.55. (174) *a.* 21.84; *b.* 16.8; *c.* $33.60; *d.* 8.4; *e.* $15.75. (175) *a.* 29.4; *b.* 22.4; *c.* $50.40; *d.* 29.61; *e.* $27.72. $2\frac{1}{2}$%: (170) *a.* 1.25; *b.* 2.2; *c.* $16.50; *d.* 2.6; *e.* $15. (171) *a.* 1.875; *b.* 4; *c.* $18; *d.* 3.2; *e.* $19.50. (172) *a.* 2.5; *b.* 6.2; *c.* $13.50; *d.* 9; *e.* $20.85. (173) *a.* 3; *b.* 8; *c.* $20; *d.* 4.5; *e.* $8.25. (174) *a.* 15.6; *b.* 12; *c.* $24; *d.* 6; *e.* $11.25. (175) *a.* 21; *b.* 16; *c.* $36; *d.* 21.15; *e.* $19.80. 7%: (170) *a.* 3.5; *b.* 6.16; *c.* $46.20; *d.* 7.28; *e.* $42. (171) *a.* 5.25; *b.* 11.2; *c.* $50.40; *d.* 8.96; *e.* $54.60. (172) *a.* 7; *b.* 17.36; *c.* $37.80; *d.* 25.2; *e.* $58.38. (173) *a.* 8.4; *b.* 22.4; *c.* $56; *d.* 12.6; *e.* $23.10. (174) *a.* 43.68; *b.* 33.6; *c.* $67.20; *d.* 16.8; *e.* $31.50. (175) *a.* 58.8; *b.* 44.8; *c.* $100.80; *d.* 59.22; *e.* $55.44. $7\frac{1}{2}$%: (170) *a.* 3.75; *b.* 6.6; *c.* $49.50; *d.* 7.8; *e.* $45. (171) *a.* 5.625; *b.* 12; *c.* $54; *d.* 9.6; *e.* $58.50. (172) *a.* 7.5; *b.* 18.6; *c.* $40.50; *d.* 27; *e.* $62.55. (173) *a.* 9; *b.* 24; *c.* $60; *d.* 13.5; *e.* $24.75. (174) *a.* 46.8; *b.* 36; *c.* $72; *d.* 18; *e.* $33.75. (175) *a.* 63; *b.* 48; *c.* $108; *d.* 63.45; *e.* $59.40. 18%: (170) *a.* 9; *b.* 15.84; *c.* $118.80; *d.* 18.72; *e.* $108. (171) *a.* 13.5; *b.* 28.8; *c.* $129.60; *d.* 23.04; *e.* $140.40. (172) *a.* 18; *b.* 44.64; *c.* $97.20; *d.* 64.8; *e.* $150.12. (173) *a.* 21.6; *b.* 57.6; *c.* $144; *d.* 32.4; *e.* $59.40. (174) *a.* 112.32; *b.* 86.4; *c.* $172.80; *d.* 43.2; *e.* $81. (175) *a.* 151.2; *b.* 115.2; *c.* $259.20; *d.* 152.28; *e.* $142.56. 23%: (170) *a.* 11.5; *b.* 20.24; *c.* $151.80; *d.* 23.92; *e.* $138. (171) *a.* 17.25; *b.* 36.8; *c.* $165.60; *d.* 29.44; *e.* $179.40. (172) *a.* 23; *b.* 57.04; *c.* $124.20; *d.* 82.8; *e.* $191.82. (173) *a.* 27.6;

b. 73.6; *c*. $184; *d*. 41.4; *e*. $75.90. (174) *a*. 143.52; *b*. 110.4; *c*. $220.80; *d*. 55.2; *e*. $103.50. (175) *a*. 193.2; *b*. 147.2; *c*. $331.20; *d*. 194.58; *e*. $182.16. 45%: (170) *a*. 22.5; *b*. 39.6; *c*. $297; *d*. 46.8; *e*. $270. (171) *a*. 33.75; *b*. 72; *c*. $324; *d*. 57.6; *e*. $351. (172) *a*. 45; *b*. 111.6; *c*. $243; *d*. 162; *e*. $375.30. (173) *a*. 54; *b*. 144; *c*. $360; *d*. 81; *e*. $148.50; (174) *a*. 280.8; *b*. 216; *c*. $432; *d*. 108; *e*. $202.50. (175) *a*. 378; *b*. 288; *c*. $648; *d*. 380.7; *e*. $356.40. 67%: (170) *a*. 33.5; *b*. 58.96; *c*. $442.20; *d*. 69.68; *e*. $402. (171) *a*. 50.25; *b*. 107.2; *c*. $482.40; *d*. 85.76; *e*. $522.60. (172) *a*. 67; *b*. 166.16; *c*. $361.80; *d*. 241.2; *e*. $558.78. (173) *a*. 80.4; *b*. 214.4; *c*. $536; *d*. 120.6; *e*. $221.10. (174) *a*. 418.08; *b*. 321.6; *c*. $643.20; *d*. 160.8; *e*. $301.50. (175) *a*. 562.8; *b*. 428.8; *c*. $964.80; *d*. 566.82; *e*. $530.64. 125%: (170) *a*. 62.5; *b*. 110; *c*. $825; *d*. 130; *e*. $750. (171) *a*. 93.75; *b*. 200; *c*. $900; *d*. 160; *e*. $975. (172) *a*. 125; *b*. 310; *c*. $675; *d*. 450; *e*. $1042.50. (173) *a*. 150; *b*. 400; *c*. $1000; *d*. 225; *e*. $412.50. (174) *a*. 780; *b*. 600; *c*. $1200; *d*. 300; *e*. $562.50. (175) *a*. 1050; *b*. 800; *c*. $1800; *d*. 1057.5; *e*. $990. 115%: (170) *a*. 57.5; *b*. 101.2; *c*. $759; *d*. 119.6; *e*. $690. (171) *a*. 86.25; *b*. 184; *c*. $828; *d*. 147.2; *e*. $897. (172) *a*. 115; *b*. 285.2; *c*. $621; *d*. 414; *e*. $959.10. (173) *a*. 138; *b*. 368; *c*. $920; *d*. 207; *e*. $379.50. (174) *a*. 717.6; *b*. 552; *c*. $1104; *d*. 276; *e*. $517.50. (175) *a*. 966; *b*. 736; *c*. $1656; *d*. 972.9; *e*. $910.80. 150%: (170) *a*. 75; *b*. 132; *c*. $990; *d*. 156; *e*. $900. (171) *a*. 112.5; *b*. 240; *c*. $1080; *d*. 192; *e*. $1170. (172) *a*. 150; *b*. 372; *c*. $810; *d*. 540; *e*. $1251. (173) *a*. 180; *b*. 480; *c*. $1200; *d*. 270; *e*. $495. (174) *a*. 936; *b*. 720; *c*. $1440; *d*. 360; *e*. $675. (175) *a*. 1260; *b*. 960; *c*. $2160; *d*. 1269; *e*. $1188. **177**. *a*. $33\frac{1}{3}$%; *b*. $33\frac{1}{3}$%; *c*. $8\frac{1}{3}$%. **178**. *a*. 300%; *b*. 300%; *c*. $6\frac{1}{4}$%. **179**. *a*. 20%; *b*. $106\frac{2}{3}$%; *c*. 50%. **180**. *a*. 500%; *b*. $93\frac{3}{4}$%; *c*. 25%.

Page 451.—**181**. *a*. 500; *b*. $160; *c*. $5600. **182**. *a*. 400; *b*. $6250; *c*. $8240. **183**. *a*. 576; *b*. $7200; *c*. $10,800. **184**. *a*. 99; *b*. $6000; *c*. $6000. **185**. *a*. $10; *b*. $33.75; *c*. $6.43. **186**. *a*. $11; *b*. $57.60; *c*. $7.10. **187**. *a*. $11; *b*. $124.20; *c*. $4.10. **188**. *a*. $8.01; *b*. $141.44; *c*. $4.03. **189**. *a*. 20% profit; *b*. $66\frac{2}{3}$% profit; *c*. 20% loss. **190**. *a*. $16\frac{2}{3}$% loss; *b*. 25% loss; *c*. 25% profit. **191**. *a*. 25% profit; *b*. 10% profit; *c*. $12\frac{1}{2}$% profit. **192**. *a*. 20% loss; *b*. $9\frac{1}{11}$% loss; *c*. 25% loss. **193**. *a*. $14, $42; *b*. $60, $60; *c*. $360, $540. **194**. *a*. $10, $70; *b*. $168, $432; *c*. $280, $720. **195**. *a*. $1.10, $2.20; *b*. $120, $360; *c*. $2625, $2375. **196**. *a*. $.90, $3.60; *b*. $135, $665; *c*. $4200, $5400.

Page 452.—**197**. *a*. $900; *b*. $187.20; *c*. $220. **198**. *a*. $600; *b*. $303; *c*. $375. **199**. *a*. $1000; *b*. $2000; *c*. $756. **200**. *a*. $1000; *b*. $1010; *c*. $1350. **201**. *a*. $12; *b*. $10. **202**. *a*. $40.64; *b*. $120. **203**. *a*. $150; *b*. $90. **204**. *a*. $20; *b*. $160. **205**. *a*. $360; *b*. $275; *c*. $49.88. **206**. *a*. $27; *b*. $2783; *c*. $81. **207**. *a*. $97.03; *b*. $940.63; *c*. $51. **208**. *a*. $150; *b*. $3000; *c*. $3700. **209**. *a*. $227.50; *b*. $3008; *c*. $12. **210**. *a*. $480; *b*. $1575; *c*. $600. **211**. *a*. $405; *b*. $22,240; *c*. $800.

Page 453.—**212**. $380; $2380. **213**. $19.37; $449.87. **214**. $423.75; $6073.75. **215**. $23.14; $693.84. **216**. $2.27; $73.22. **217**. $5.75; $69.17. **218**. $103.50; $944.40. **219**. $362.05; $5682.05. **220**. $297.74; $6547.74. **221**. $76.81; $652.61. **222**. $244.75; $9144.75. **223**. $104.33; $754.73. **224**. July 2; 5 mo.; $50; $2350. **225**. June 1; 78 da.; $41.60; $3158.40. **226**. Aug. 3; 3 mo.; $70; $5530. **227**. Sept. 6; 53 da.; $44.17;

$7455.83. **228.** July 30; 76 da.; $63.33; $4936.67. **229.** July 31; 46 da.; $38.33; $5961.67. **230.** Sept. 14; 13 da.; $13.90; $6986.10. **231.** March 7; 173 da.; $245.08; $8254.92. **232.** March 1; 137 da.; $144.61; $9355.39. **233.** April 1; 107 da.; $148.61; $9851.39.

Page 454. — **234.** *a.* $8850; *b.* $595.63; *c.* $4425. **235.** *a.* $48,500; *b.* $1030; *c.* $10,362.50. **236.** *a.* $1312.50; *b.* $2492.50; *c.* $19,400. **237.** *a.* $1006.25; *b.* $3587.50; *c.* $102,375. **238.** (234, *c*) $250; 5.64%. (235, *c*) $400; 3.86%. (236, *c*) $700; 3.6%. (237, *c*) $4000; 3.9%. **239.** *a.* 169; *b.* 289; *c.* 484; *d.* 15,625; *e.* 190,096. **240.** *a.* 196; *b.* 324; *c.* 529; *d.* 10,000; *e.* 323,761. **241.** *a.* 225; *b.* 361; *c.* 576; *d.* 250,000; *e.* 515,524. **242.** *a.* 100; *b.* 441; *c.* 625; *d.* 360,000; *e.* 422,500. **243.** *a.* 256; *b.* 1296; *c.* 2401; *d.* 11,236; *e.* 518,400. **244.** *a.* 24; *b.* 18; *c.* 72; *d.* 105. **245.** *a.* 23; *b.* 32; *c.* 55; *d.* 109. **246.** *a.* 21; *b.* 45; *c.* 67; *d.* 221. **247.** *a.* 19; *b.* 99; *c.* 73; *d.* 336. **248.** *a.* 22; *b.* 100; *c.* 85; *d.* 245. **249.** *a.* 17; *b.* 93; *c.* 88; *d.* 364.

Page 455. — **1.** Smith, $74.93; Doe, $41.88; Dean, $74.85; King, $35.86; Rula, $69.70; Riley, $84.24; Gimlin, $56.13; Johns, $28.38; Lund, $59.55; Bell, $33.02.

Page 456 — **2.** $714.36. **3.** 4.08%. **4.** $67.46. **5.** $78.04.

Page 457. — **6.** 161.87 lb.; $54.39. **7.** 222.81 lb.; $76.65. **8.** 234.66 lb.; $78.85. **9.** 266.19 lb.; $87.31. **10.** 255.87 lb.; $89.04. **11.** 184.87 lb.; $60.45. **12.** 229.53 lb.; $75.06. **13.** 162.82 lb.; $53.57. **14.** 291.32 lb.; $96.72. **15.** 275.94 lb.; $93.82. **16.** 61,121 lb. **17.** 2285.88 lb. **18.** $765.86. **19.** 6112.1 lb. **20.** Nos. 4, 5, 7, 9, 10. **21.** 3.725%. **22.** Nos. 2, 3, 4, 9, 10. **23.** Nos. 4, 9, 10. **24.** 228.58 lb.

Page 458. — **25.** No. 1, $33.20; No. 2. $39.84; No. 3, $39.84; No. 4, $39.47; No. 5, $39.84; No. 6, $36.40; No. 7, $37.20; No. 8, $36.40; No. 9, $36.77; No. 10, $39.84. **26.** No. 1, $28.92; No. 2, $45.59; No. 3, $48.44; No. 4, $58.73; No. 5, $60.35; No. 6, $32.20; No. 7, $48.69; No. 8, $26.06; No. 9, $71.93; No. 10, $63.92. **27.** $484.83. **28.** $48.48. **29.** Nos. 4, 5, 7, 9, 10. **30.** No. 1, 20.5¢; $1.64. **31.** No. 2, 17.8¢; $1.93. **32.** No. 3, 16.9¢; $1.99. **33.** No. 4. 14.8¢; $2.21. **34.** No. 5, 15.5¢; $2.24. **35.** No. 6, 19.6¢; $1.67. **36.** No. 7, 16.2¢; $2.02. **37.** No. 8, 22.3¢; $1.47. **38.** No. 9, 12.6¢; $2.63. **39.** No. 10, 14.4¢; $2.36. **40.** 17¢. **41.** Nos. 3, 4, 5, 7, 9, 10.

Page 459. — **42.** No. 1, 138.17 lb.; No. 2, 86.25 lb.; No. 3, 102.98 lb.; No. 4, 203.92 lb.; No. 5, 194.65 lb.; No. 6, 275.57 lb.; No. 7, 110.5 lb.; No. 8, 70.33 lb.; No. 9, 182.11 lb.; No. 10, 225.44 lb. **43.** 1589.92 lb. **44.** No. 1, $47.81; No. 2, $25.01; No. 3, $31.61; No. 4, $68.11; No. 5, $62.29; No. 6, $92.87; No. 7, $34.03; No. 8, $21.94; No. 9, $63.56; No. 10, $76.88. **45.** No. 1, $30.10; No. 2, $23; No. 3, $24.38; No. 4, $24.38; No. 5. $24.38; No. 6, $24.38; No. 7, $24.38; No. 8, $24.38; No. 9, $33.30; No. 10, $33.30. **46.** No. 1, $23.39; No. 2, $5.38; No. 3, $11.52; No. 4, $52.03; No. 5, $44.97; No. 6, $79.62; No. 7, $14.21; No. 8, $.51; No. 9, $37.91; No. 10, $52.57. **47.** No. 1, 21.7¢; No. 2, 26.6¢; No. 3, 23.6¢; No. 4, 11.9¢; No. 5, 12.5¢; No. 6, 8.8¢; No. 7, 22¢; No. 8, 34.6¢; No. 9, 18.2¢; No. 10, 14.7¢. **48.** No. 1, $1.59; No. 2, $1.09; No. 3, $1.30; No. 4, $2.79; No. 5, $2.55; No. 6, $3.80; No. 7, $1.40; No. 8, $.90; No. 9, $1.91; No. 10, $2.31. **49.** Nos. 4, 5, 6, 9, 10; Nos. 4, 5, 6, 9, 10.

Page 460. — **50.** \$.214. **51.** \$73.71. **52.** \$183.19. **53.** \$21.79. **54.** \$156.06. **55.** \$26.25. **56.** \$65.54. **57.** \$167.62. **58.** \$19.67. **59.** \$139.61. **60.** \$23.25. **61.** \$45.61. **62.** \$126.73. **63.** \$.11. **64.** \$106.78. **65.** \$4.62.

Page 461. — **66.** 125.75%. **67.** 175.94%. **68.** .46%. **69.** 162.45%. **70.** 18.75%. **71.** 5.04%. **72.** 6.47%. **73.** 5.69%. **74.** 5.6%. **75.** 4.89%. **76.** \$31.50. **77.** \$73.50. **78.** \$59.50. **79.** 192 lb. **80.** \$19.95. **81.** \$187.20 gain.

Page 462. — **82.** \$.784 a bushel. **83.** \$5.49 per hundred. **84.** \$2.29. **85.** 5046 lb. **86.** \$.16$\frac{1}{14}$. **87.** \$504.

Page 463. — **88.** \$35.60. **89.** \$.25$\frac{3}{7}$. **90.** \$.02$\frac{1}{2}$. **91.** \$.04$\frac{2}{7}$. **92.** \$.03$\frac{4}{7}$. **93.** \$.07$\frac{1}{14}$. **94.** \$.43$\frac{1}{11}$. **95.** \$63.80; \$.46.

Page 464. — **96.** (1) \$222; (2) \$225; (3) \$200; (4) \$140; (5) \$137.50; (6) \$125; (7) \$118.25; (8) \$112.50; (9) \$80; (10) \$84; (11) \$76; (12) \$79.20; (13) \$200; (14) \$130.91; (15) \$100; (16) \$80; (17) \$125; (18) \$100; (19) \$125; (20) \$160. **97.** (1) \$190.90; (2) \$187.10; (3) \$161; (4) \$107.70; (5) \$95.50; (6) \$94.50; (7) \$77.65; (8) \$76.40; (9) \$44.75; (10) \$55.50; (11) \$48.20; (12) \$45.50; (13) \$160.70; (14) \$100.31; (15) \$69.75; (16) \$49; (17) \$91; (18) \$51.55; (19) \$102.05; (20) \$130. **98.** (1) \$1.27; (2) \$.75; (3) \$.81; (4) \$.77; (5) \$.35; (6) \$.38; (7) \$.36; (8) \$.34; (9) \$.56; (10) \$.40; (11) \$.48; (12) \$.26; (13) \$1.29; (14) \$.41; (15) \$.69; (16) \$.49; (17) \$.36; (18) \$.21; (19) \$.82; (20) \$.41. **99.** (1) \$.21; (2) \$.15; (3) \$.20; (4) \$.23; (5) \$.15; (6) \$.13; (7) \$.19; (8) \$.16; (9) \$.44; (10) \$.20; (11) \$.28; (12) \$.19; (13) \$.31; (14) \$.12; (15) \$.30; (16) \$.31; (17) \$.14; (18) \$.19; (19) \$.18; (20) \$.09. **100.** \$34.065. **101.** 185.9 bu. per acre.

Page 465. — **102.** 57$\frac{1}{2}$ T. **103.** 78 T. **104.** \$19.23. **105.** 88 T. **106.** 29$\frac{1}{6}$ ft. **107.** 62$\frac{1}{2}$ ft. **108.** \$36.48. **109.** \$116.40.

Page 466. — **110.** \$349.20. **111.** \$83.20. **112.** \$12. **113.** 72 T. **114.** 20.1 A. **115.** 36. **116.** 31.8 ft. **117.** 20.1 da. **118.** 42 cows.

Page 467. — **119.** 28,800,000 lb. **120.** 3.17 in. **121.** 118,200 cu. ft. **122.** 40,474.5 T. **123.** 4.46 in.; 20,237.25 T. **124.** 11,220,000 lb.

Page 468. — **126.** \$848. **127.** $\frac{5}{6}$. **128.** $\frac{17}{20}$. **129.** 36$\frac{4}{11}$ bu. **130.** 55.2¢. **131.** 63$\frac{17}{46}$¢. **132.** \$1.08. **133.** \$11.50 gain.

Page 469. — **134.** \$.10 gain. **135.** 168 bu.; 210 bu. **136.** 302$\frac{2}{5}$ bu.; 378 bu. **137.** 22$\frac{2}{28}$ ft.; 312$\frac{1}{2}$ bu. **138.** \$20.80. **139.** \$21.60. **140.** \$19.60.

Page 470. — **1.** 316.38%. **2.** 1176.05%. **3.** 3800 lb. **4.** 28.62%. **5.** 11.23%. **6.** 967,233,991 T.; 8.84% agriculture; 2.45% animals; 55.75% mines; 11.21% forests; 13.97% manufactures; 3.77% merchandise; 3.97% other commodities. **7.** \$896,705,984; 33.96% iron and steel; 15.63% copper; 4.52% agricultural implements; 12.9% wood manufactures; 14.46% mineral oils; 2.96% chemicals; 7.12% leather; 5.99% cotton; 1.12% books; 1.3% paper.

Page 475. — **4.** West; 22° 30′. **5.** 7 min. 20 sec.

Page 476. — **7.** 2 hr. 40 min. **8.** 59 min. 20 sec. **9.** 5 hr. 8 min. $12\frac{2}{5}$ sec. **10.** 40 min. 55 sec. past 10 A.M. **11.** 26 min. $55\frac{3}{5}$ sec. past 10 P.M., April 30, 1898.

Page 477. — **13.** 13° 15′. **14.** 10° 23′. **15.** East; 68° 10′ W. long. **16.** 47 min. 4 sec. past 11 A.M.

Page 480. — **2.** \$131.67. **3.** \$206.72. **4.** \$661.44. **5.** \$1365.90. **6.** \$2661.78. **7.** \$2922.15. **8.** \$1324.25. **9.** \$719.81.
2. \$12.50. **3.** \$6.67. **4.** \$37.50. **5.** \$54.

Page 482. — **2.** \$1609.76. **3.** \$5518.06. **4.** \$1647.34. **5.** \$456.96. **6.** \$7555.35. **7.** \$11,940.52.

Page 484. — **2.** \$465.50. **3.** \$5360. **4.** \$137.68.

Page 486. — **2.** 57.

Page 488. — **2.** 32. **3.** 43. **4.** 85. **5.** 7.2. **6.** .17.

Page 492. — **2.** 46. **3.** 53.

Page 496. — **7.** 18 da. **8.** $24\frac{8}{10}$ da. **9.** 54 da.

Page 497. — **4.** \$974. **5.** \$967.12. **6.** \$2378.13. **7.** \$2442.93. **8.** 1766.355 taels.

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